

Exercise walk-through

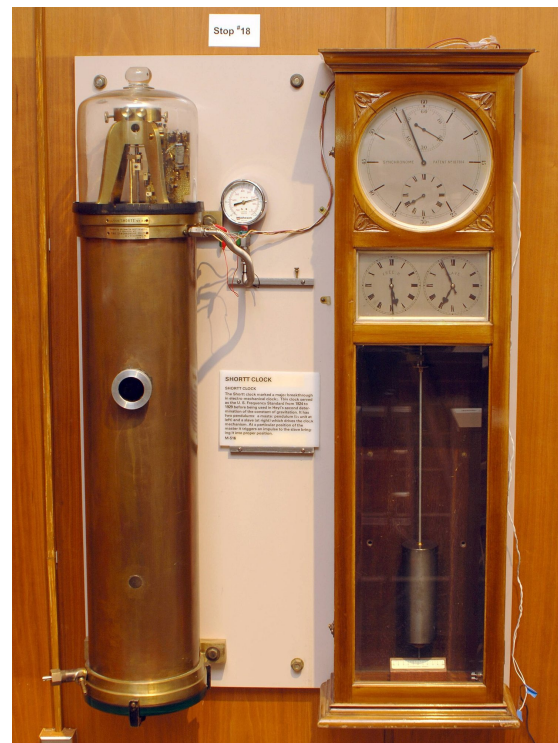
17.1

Simple harmonic oscillations

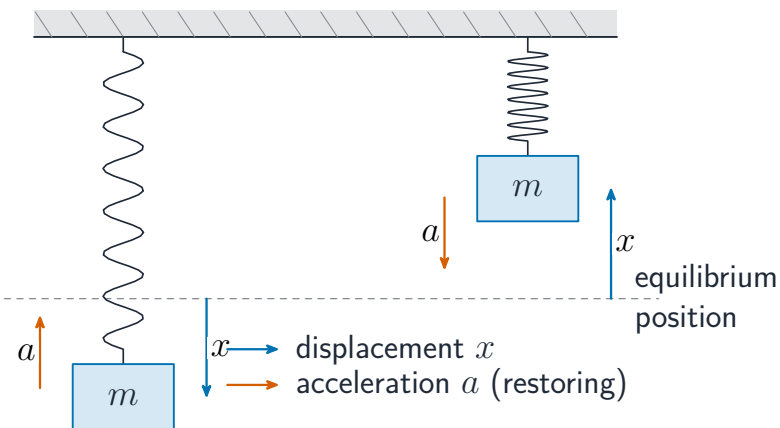
A-Level Physics

What we will cover

- 1 SHM: $a = -\omega^2 x$
- 2 Displacement, velocity, acceleration
- 3 Graphs
- 4 Energy
- 5 Damping & resonance
- 6 Recap



Simple harmonic motion



A body moves with **simple harmonic motion** when its acceleration is

- **proportional to its displacement** from a fixed **equilibrium** point, and
- **directed back towards that point.**

Many systems do this near a stable equilibrium: a mass on a **spring**, a **pendulum** (small swing), a floating block, the charge on a **capacitor** in an LC circuit, atoms in a solid.

The words you must use precisely

displacement x – from equilibrium, signed

amplitude x_0 – the largest $|x|$, always +

period T – one full oscillation

frequency $f = 1/T$, in Hz

angular frequency
 $\omega = 2\pi f = 2\pi/T$

phase difference – a fraction of a cycle, in rad

Given any one of ω , f , T you can reach the other two – start every question by getting ω .

Exercises 2.1·2.3

2.1 State the **two** conditions for simple harmonic motion. [2]

2.2 Define the **amplitude** of an oscillation. [1]

2.3 A particle has $\omega = 5.0 \text{ rad s}^{-1}$ and amplitude 0.040 m. Find its **maximum speed**. [2]

Exercises 2.1-2.3

2.1 State the **two** conditions for simple harmonic motion. [2]

2.1

- acceleration is **proportional to displacement** from a fixed point
- and **directed towards** that point (opposite to displacement)

2.2 Define the **amplitude** of an oscillation. [1]

2.2

- the **maximum displacement** from the equilibrium position

2.3 A particle has $\omega = 5.0 \text{ rad s}^{-1}$ and amplitude 0.040 m. Find its **maximum speed**. [2]

2.3

- $v_{\text{max}} = \omega x_0$

$$v_{\text{max}} = \omega x_0 = (5.0 \text{ rad s}^{-1})(0.040 \text{ m}) = \mathbf{0.20} \frac{\text{m}}{\text{s}}$$

Exercises 2.4·3.1

2.4 State the **phase relationship** between displacement and acceleration in SHM. [1]

3.1 In simple harmonic motion, the acceleration is: [1]

☐

A proportional to displacement and in the same direction

B proportional to displacement and in the opposite direction

C constant

D always zero

Exercises 2.4·3.1

2.4 State the **phase relationship** between displacement and acceleration in SHM. [1]

2.4

- they are **antiphase** (phase difference π / 180°)

3.1 In simple harmonic motion, the acceleration is: [1]

☐

A proportional to displacement and in the same direction

B proportional to displacement and in the opposite direction

C constant

D always zero

3.1 B

- SHM: acceleration is **proportional to displacement and opposite in direction**

Exercise 3.2

3.2 A mass on a spring oscillates with period 0.50 s and amplitude 0.030 m. Find (a) the angular frequency, (b) the maximum speed, (c) the maximum acceleration. [4]

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3.2

(a) $\omega = 2\pi/T$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.50 \text{ s}} = \mathbf{12.6 \text{ rad s}^{-1}}$$

(b) $v_{\max} = \omega x_0 = (12.57 \text{ rad s}^{-1})(0.030 \text{ m}) = \mathbf{0.38 \frac{m}{s}}$

(c) $a_{\max} = \omega^2 x_0$

$$a_{\max} = (12.57 \text{ rad s}^{-1})^2(0.030 \text{ m}) = \mathbf{4.7 \text{ m s}^{-2}}$$

Exercises 3.3-3.4

3.3 A particle in SHM has amplitude 0.10 m and $\omega = 8.0 \text{ rad s}^{-1}$. Find its **speed** when $x = 0.060 \text{ m}$. [3]

3.4 Sketch the **displacement–time** and **acceleration–time** graphs for SHM, and state their phase relationship. [3]

Exercises 3.3·3.4

3.3 A particle in SHM has amplitude 0.10 m and $\omega = 8.0 \text{ rad s}^{-1}$. Find its **speed** when $x = 0.060 \text{ m}$. [3]

3.3

$$\bullet \quad v = \omega \sqrt{x_0^2 - x^2}$$

$$v = \omega \sqrt{x_0^2 - x^2} = (8.0 \text{ rad s}^{-1}) \sqrt{(0.10 \text{ m})^2 - (0.060 \text{ m})^2} = (8.0 \text{ rad s}^{-1}) \sqrt{0.0064 \text{ m}^2}$$

$$v = \omega \sqrt{x_0^2 - x^2} = (8.0 \text{ rad s}^{-1})(0.080 \text{ m}) = \mathbf{0.64} \frac{\text{m}}{\text{s}}$$

3.4 Sketch the **displacement–time** and **acceleration–time** graphs for SHM, and state their phase relationship. [3]

3.4

- displacement is a **sine** (or cosine) curve
- acceleration is the **same shape inverted** (negative)
- they are **antiphase** (180° out of phase)

Exercise 3.5 (a)

3.5 An object oscillates. A graph of its acceleration a against displacement x is a straight line through the origin passing through $(0.05 \text{ m}, -8.0 \text{ m s}^{-2})$.

(a) Explain how this graph shows the motion is **simple harmonic**.

[2]

Exercise 3.5 (a)

3.5 An object oscillates. A graph of its acceleration a against displacement x is a straight line through the origin passing through $(0.05 \text{ m}, -8.0 \text{ m s}^{-2})$.

(a) Explain how this graph shows the motion is **simple harmonic**. [2]

(a) the line is **straight and through the origin**, so a is **proportional to x**

- its **negative gradient** shows a is in the **opposite direction** to x – i.e. $a = -\omega^2 x$, the SHM condition

Exercise 3.5 (b)

3.5 An object oscillates. A graph of its acceleration a against displacement x is a straight line through the origin passing through $(0.05 \text{ m}, -8.0 \text{ m s}^{-2})$.

(b) Use the **gradient** to find the angular frequency ω and the period T . [3]

Exercise 3.5 (b)

3.5 An object oscillates. A graph of its acceleration a against displacement x is a straight line through the origin passing through $(0.05 \text{ m}, -8.0 \text{ m s}^{-2})$.

(b) Use the **gradient** to find the angular frequency ω and the period T . [3]

$$\text{(b) gradient} = -8.0 \text{ m s}^{-2} / 0.05 \text{ m} = -160 \text{ s}^{-2} = -\omega^2$$

$$\omega = \sqrt{-\text{gradient}} = \sqrt{160 \text{ s}^{-2}} = 12.6 \text{ rad s}^{-1}$$

$$T = \frac{2\pi}{\omega} = \mathbf{0.50 \text{ s}}$$

Exercise 3.6 (a)

3.6 A small ball hangs on a spring and is set oscillating vertically. A motion sensor records its displacement, and the graph obtained is a smooth curve whose successive maximum displacements are equal, with a maximum displacement of x_0 and 8 complete oscillations in 6.4 s.

(a) A second graph plots the ball's **acceleration** against its **displacement** and gives a **straight line through the origin with a negative gradient**. **Explain** how this shows that the motion is simple harmonic. [2]

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(a) A second graph plots the ball's **acceleration** against its **displacement** and gives a **straight line through the origin with a negative gradient**. **Explain** how this shows that the motion is simple harmonic. [2]

(a) a straight line through the origin shows $a \propto x$

- the negative gradient shows the acceleration is always directed **opposite** to the displacement, i.e. towards the equilibrium position – which is the definition of SHM

Exercise 3.6 (b)

3.6 A small ball hangs on a spring and is set oscillating vertically. A motion sensor records its displacement, and the graph obtained is a smooth curve whose successive maximum displacements are equal, with a maximum displacement of x_0 and 8 complete oscillations in 6.4 s.

(b) **Determine** the period T and the angular frequency ω of the oscillations. [3]

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3.6 A small ball hangs on a spring and is set oscillating vertically. A motion sensor records its displacement, and the graph obtained is a smooth curve whose successive maximum displacements are equal, with a maximum displacement of x_0 and 8 complete oscillations in 6.4 s.

(b) **Determine** the period T and the angular frequency ω of the oscillations. [3]

(b) $T = 6.4 \text{ s} / 8 = 0.80 \text{ s}$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.80 \text{ s}} = \mathbf{7.9 \text{ rad s}^{-1}}$$

Exercise 3.6 (c)

3.6 A small ball hangs on a spring and is set oscillating vertically. A motion sensor records its displacement, and the graph obtained is a smooth curve whose successive maximum displacements are equal, with a maximum displacement of x_0 and 8 complete oscillations in 6.4 s.

(c) The magnitude of the gradient of that acceleration-displacement line is 61 s^{-2} . **Show that** this is consistent with your value of ω . [2]

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(c) The magnitude of the gradient of that acceleration-displacement line is 61 s^{-2} . **Show that** this is consistent with your value of ω . [2]

(c) for SHM $a = -\omega^2 x$, so the magnitude of the gradient is ω^2

$$\omega = \sqrt{|\text{gradient}|} = \sqrt{61 \text{ s}^{-2}} = 7.8 \text{ rad s}^{-1}$$

which agrees with (b)

Exercise 3.6 (d)

3.6 A small ball hangs on a spring and is set oscillating vertically. A motion sensor records its displacement, and the graph obtained is a smooth curve whose successive maximum displacements are equal, with a maximum displacement of x_0 and 8 complete oscillations in 6.4 s.

(d) **State** what is meant by **damping**.

[2]

Exercise 3.6 (d)

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(d) **State** what is meant by **damping**. [2]

(d) damping is the loss of energy from an oscillating system to its surroundings, usually through **resistive forces**

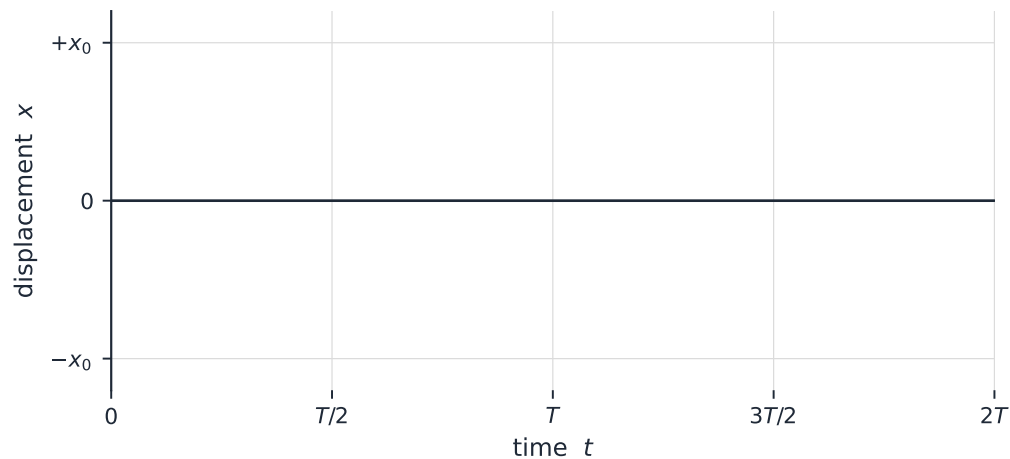
- so the **amplitude decreases** with time

Setup for exercise 3.6

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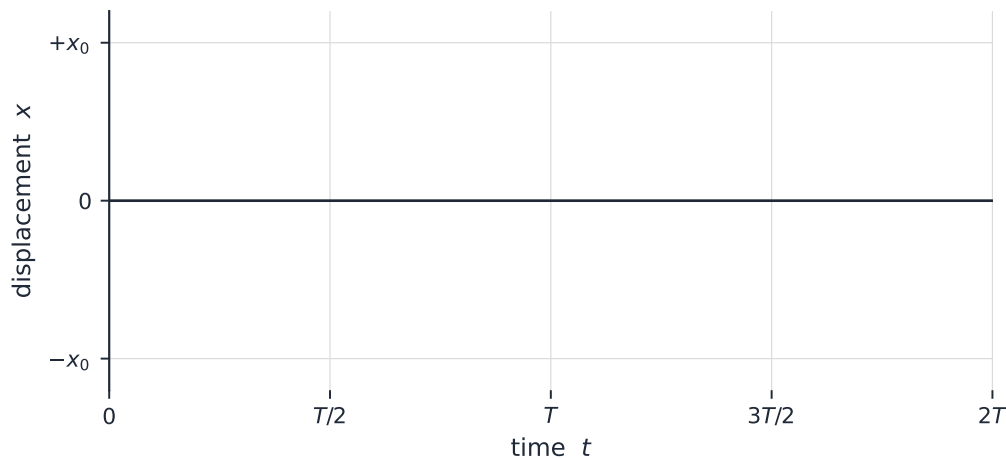
Exercise 3.6 (e)

(e) The ball is now made to oscillate in a beaker of oil, and is released from the same displacement x_0 . On the axes below, **sketch** a possible variation of its displacement with time from $t = 0$ to $t = 2T$. [3]



Exercise 3.6 (e)

(e) The ball is now made to oscillate in a beaker of oil, and is released from the same displacement x_0 . On the axes below, **sketch** a possible variation of its displacement with time from $t = 0$ to $t = 2T$. [3]



(e) a curve starting at $+x_0$

- oscillating with the **same period** T so that it still completes two cycles by $2T$
- with each maximum displacement **smaller** than the one before, decaying smoothly