

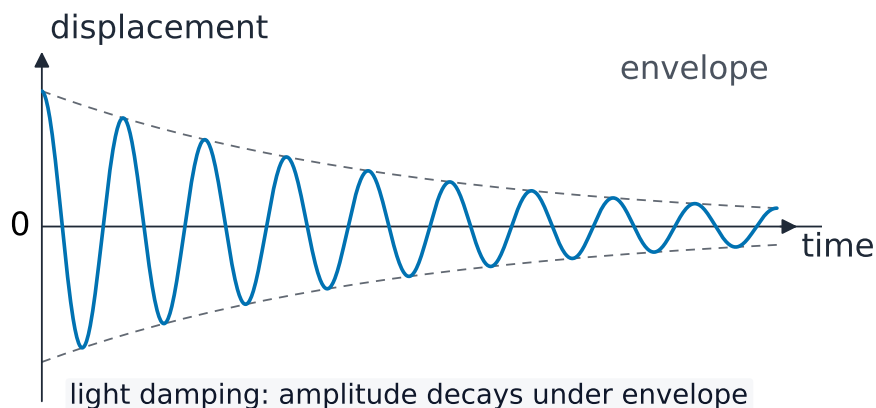
Exercise walk-through

17.1.1

Proving a system is simple harmonic, and reading phase from a graph

A-Level Physics

Damped oscillations

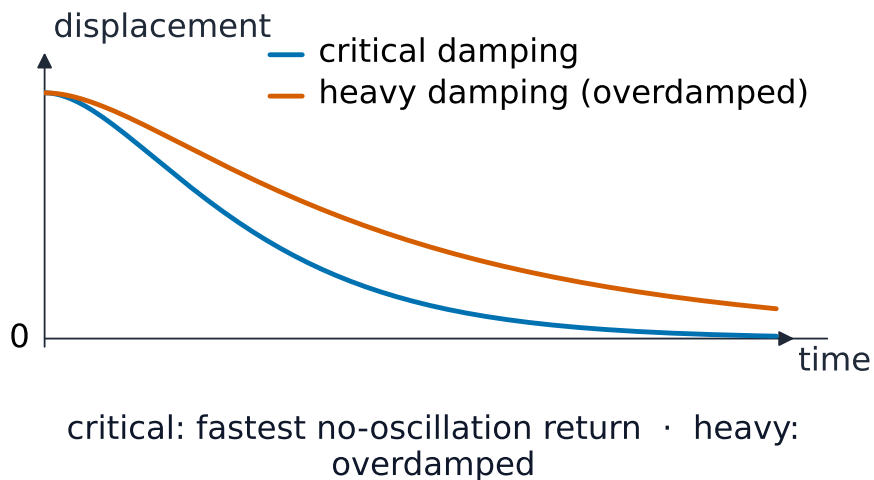


A resistive force (**air resistance**, **friction**) removes energy as heat, so the amplitude **shrinks**. **Light**

damping : the amplitude dies away slowly over **many** cycles, still near the natural frequency.

A car's suspension is light-to-medium damped – bumps die away, but the ride stays smooth.

Critical and heavy damping



- **critical damping** : back to equilibrium in the **shortest** time, **no** overshoot, no oscillation
- **heavy damping** : no oscillation either, but **slower** than critical

A **galvanometer** needle is **critically** damped so it settles at once; a strong door closer is **heavily** damped.

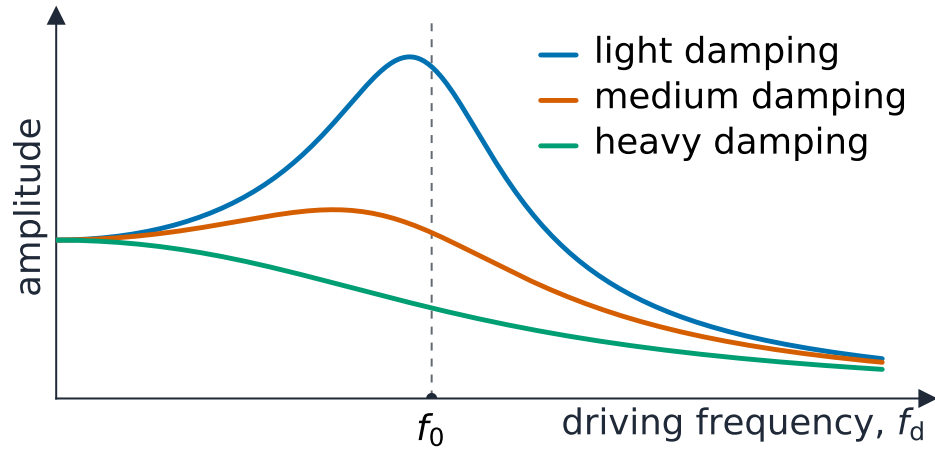
Forced oscillations and resonance

A **forced oscillation** runs at the **driving** frequency f_d , not its own.

Resonance : $f_d = f_0$, the **natural frequency** – **largest** amplitude, most efficient transfer.

Damping shapes the **resonance curve** : **lighter** → sharper and higher; **heavier** → broader, lower, and nudged to a lower frequency.

Forced oscillations and resonance, in detail



resonance: max amplitude near f_0 · lighter damping
⇒ sharper peak

Forced oscillations and resonance A forced oscillation runs at the driving frequency f_d , not its own.

Resonance in the world



- a **swing** pushed at the right rate builds a big amplitude
- a **wine glass** shattered by its own ringing note
- a **building** shaken by an earthquake at a matching frequency

Engineers design a building's natural frequencies to **avoid** the main earthquake range – resonance is as often something to dodge as to exploit.

Exercise 2.1

2.1 State the condition for simple harmonic motion in words, and write it as an equation. [2]

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2.1

- the **acceleration is proportional to the displacement** from a fixed point, and is always directed **towards** that point
- $a = -\omega^2 x$

Exercise 2.2

2.2 A mass m hangs on a spring of spring constant k and is displaced a distance x from equilibrium. Show that $a = -\frac{k}{m}x$, and hence write down the period. [3]

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2.2

- the restoring force from the spring is $F = -kx$
- apply Newton's second law

$$a = \frac{F}{m} = -\frac{k}{m}x$$

- comparing with $a = -\omega^2 x$ gives $\omega^2 = \frac{k}{m}$, so

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

Exercises 2.3·2.4

2.3 Explain why the oscillation of a simple pendulum is only **approximately** simple harmonic. [2]

2.4 A particle performs simple harmonic motion with amplitude 24 mm and period 0.80 s. Calculate its maximum acceleration. [3]

Exercises 2.3·2.4

2.3 Explain why the oscillation of a simple pendulum is only **approximately** simple harmonic. [2]

2.3

- the restoring force is $-mg \sin \theta$, which is proportional to $\sin \theta$ and **not** to the displacement itself
- only for **small angles** does $\sin \theta \approx \theta$, making the force proportional to x ; for a large amplitude the approximation fails and the motion is no longer simple harmonic

2.4 A particle performs simple harmonic motion with amplitude 24 mm and period 0.80 s. Calculate its maximum acceleration. [3]

2.4

- find ω from the period

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.80 \text{ s}} = 7.85 \text{ rad s}^{-1}$$

- then $a_0 = \omega^2 x_0$, with the amplitude in **metres**

$$a_0 = \omega^2 x_0 = (7.85 \text{ rad s}^{-1})^2 \times 0.024 \text{ m} = \mathbf{1.5 \text{ m s}^{-2}}$$

Exercises 2.5-2.6

2.5 Two oscillators have the same period of 0.60 s. Corresponding peaks on their displacement-time graphs are 0.15 s apart. Calculate the phase difference, in radians. [2]

2.6 State the phase difference between displacement and velocity, and between displacement and acceleration, for a body in simple harmonic motion. [2]

Exercises 2.5·2.6

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2.5

- the lag is a fraction $\frac{\Delta t}{T} = \frac{0.15 \text{ s}}{0.60 \text{ s}} = \frac{1}{4}$ of a cycle

$$\varphi = 2\pi \times \frac{1}{4} = \frac{\pi}{2} \text{ rad (1.6 rad)}$$

2.6 State the phase difference between displacement and velocity, and between displacement and acceleration, for a body in simple harmonic motion. [2]

2.6

- displacement and velocity: $\frac{\pi}{2}$ rad, with the velocity **ahead**
- displacement and acceleration: π rad – exactly out of phase

Exercise 3.1

3.1 For a body in simple harmonic motion, a graph of acceleration a against displacement x is a straight line through the origin. The angular frequency ω is found from the gradient as [1]

☐

A $\omega = \text{gradient}$

B $\omega = -\text{gradient}$

C $\omega = \sqrt{-\text{gradient}}$

D $\omega = \sqrt{\text{gradient}}$

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3.1 C

- $a = -\omega^2 x$, so the gradient of the line is $-\omega^2$, which is negative
- therefore $\omega = \sqrt{-\text{gradient}}$. **A** and **B** forget the square root; **D** gives the square root of a negative number

Exercise 3.2 (a)

3.2 A uniform cylinder of cross-sectional area A and mass m floats upright in a liquid of density ρ . It is pushed vertically downwards a distance x and released.

(a) Show that the resultant force on the cylinder is $-\rho Agx$. [2]

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3.2 A uniform cylinder of cross-sectional area A and mass m floats upright in a liquid of density ρ . It is pushed vertically downwards a distance x and released.

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(a)

- at equilibrium the upthrust already balances the weight, so the resultant force comes only from the **extra** upthrust
- the extra volume submerged is Ax , so the extra upthrust is ρAxg , acting **upwards** while x is measured **downwards**

$$F = -\rho Agx$$

Exercise 3.2 (b)

3.2 A uniform cylinder of cross-sectional area A and mass m floats upright in a liquid of density ρ . It is pushed vertically downwards a distance x and released.

(b) Hence show that the motion is simple harmonic, and give an expression for ω^2 . [2]

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3.2 A uniform cylinder of cross-sectional area A and mass m floats upright in a liquid of density ρ . It is pushed vertically downwards a distance x and released.

(b) Hence show that the motion is simple harmonic, and give an expression for ω^2 . [2]

(b)

- apply Newton's second law

$$a = \frac{F}{m} = - \left(\frac{\rho Ag}{m} \right) x$$

- ρ , A , g and m are constant, so $a \propto -x$: the motion is simple harmonic, with

$$\omega^2 = \frac{\rho Ag}{m}$$

Exercise 3.2 (c)

3.2 A uniform cylinder of cross-sectional area A and mass m floats upright in a liquid of density ρ . It is pushed vertically downwards a distance x and released.

(c) A cylinder of mass 0.45 kg and cross-sectional area $2.8 \times 10^{-3} \text{ m}^2$ floats in water of density 1000 kg m^{-3} . Calculate its period of oscillation. [2]

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(c)

- substitute, keeping units

$$\omega^2 = \frac{\rho A g}{m} = \frac{1000 \text{ kg m}^{-3} \times 2.8 \times 10^{-3} \text{ m}^2 \times 9.81 \text{ m s}^{-2}}{0.45 \text{ kg}} = 61.0 \text{ s}^{-2}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{7.81 \text{ rad s}^{-1}} = \mathbf{0.80 \text{ s}}$$

Exercise 3.3 (a)

3.3 A student measured the period T of a simple pendulum for several lengths L and plotted T^2 against L . The graph was a straight line through the origin with gradient $4.06 \text{ s}^2 \text{ m}^{-1}$.

(a) Show that $T^2 = \frac{4\pi^2}{g}L$ for a simple pendulum.

[2]

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(a) Show that $T^2 = \frac{4\pi^2}{g}L$ for a simple pendulum. [2]

(a)

- for a small angle the restoring force is $-mg\frac{x}{L}$, so $a = -\frac{g}{L}x$ and $\omega^2 = \frac{g}{L}$
- therefore

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}} \implies T^2 = \frac{4\pi^2}{g}L$$

Exercise 3.3 (b)

3.3 A student measured the period T of a simple pendulum for several lengths L and plotted T^2 against L . The graph was a straight line through the origin with gradient $4.06 \text{ s}^2 \text{ m}^{-1}$.

(b) Use the gradient to calculate a value for g .

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(b) Use the gradient to calculate a value for g . [2]

(b)

- the gradient is $\frac{4\pi^2}{g}$, so rearrange for g

$$g = \frac{4\pi^2}{\text{gradient}} = \frac{4\pi^2}{4.06 \text{ s}^2 \text{ m}^{-1}} = \mathbf{9.7 \text{ m s}^{-2}}$$

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3.3 A student measured the period T of a simple pendulum for several lengths L and plotted T^2 against L . The graph was a straight line through the origin with gradient $4.06 \text{ s}^2 \text{ m}^{-1}$.

(c) State **two** assumptions made in deriving the expression in (a). [2]

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(c)

- any **two** of: the angle of swing is **small**, so that $\sin \theta \approx \theta$; the mass of the **string** is **negligible**; the bob behaves as a **point mass**; **air resistance** and other damping are negligible; the string does not stretch

Exercise 3.3 (d)

3.3 A student measured the period T of a simple pendulum for several lengths L and plotted T^2 against L . The graph was a straight line through the origin with gradient $4.06 \text{ s}^2 \text{ m}^{-1}$.

(d) The pendulum is set to a length of 0.65 m and released from a horizontal displacement of 35 mm. Calculate the maximum speed of the bob. [3]

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(d) The pendulum is set to a length of 0.65 m and released from a horizontal displacement of 35 mm . Calculate the maximum speed of the bob. [3]

(d)

- find the period from the graph, then ω

$$T^2 = \text{gradient} \times l = 4.06 \text{ s}^2 \text{ m}^{-1} \times 0.65 \text{ m} = 2.64 \text{ s}^2, \quad T = 1.62 \text{ s}$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{1.62 \text{ s}} = 3.87 \text{ rad s}^{-1}$$

- then $v_0 = \omega x_0$, with the amplitude in metres

$$v_0 = \omega x_0 = 3.87 \text{ rad s}^{-1} \times 0.035 \text{ m} = \mathbf{0.14} \frac{\text{m}}{\text{s}}$$

Exercise 3.3 (e)

3.3 A student measured the period T of a simple pendulum for several lengths L and plotted T^2 against L . The graph was a straight line through the origin with gradient $4.06 \text{ s}^2 \text{ m}^{-1}$.

(e) Calculate the speed of the bob when its displacement is 20 mm. [2]

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(e) Calculate the speed of the bob when its displacement is 20 mm. [2]

(e)

- use $v = \omega \sqrt{x_0^2 - x^2}$

$$v = \omega \sqrt{x_0^2 - x^2} = 3.87 \text{ rad s}^{-1} \times \sqrt{(0.035 \text{ m})^2 - (0.020 \text{ m})^2}$$

$$v = \omega \sqrt{x_0^2 - x^2} = 3.87 \text{ rad s}^{-1} \times 0.0287 \text{ m} = \mathbf{0.11} \frac{\text{m}}{\text{s}}$$

Exercise 3.3 (f)

3.3 A student measured the period T of a simple pendulum for several lengths L and plotted T^2 against L . The graph was a straight line through the origin with gradient $4.06 \text{ s}^2 \text{ m}^{-1}$.

(f) State and explain the effect on the period of doubling the mass of the bob. [1]

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(f) State and explain the effect on the period of doubling the mass of the bob. [1]

(f)

- **no change:** the mass cancelled in $a = -\frac{g}{L}x$, so the period depends only on L and g