

Exercise walk-through

# 15.3

## Kinetic theory of gases

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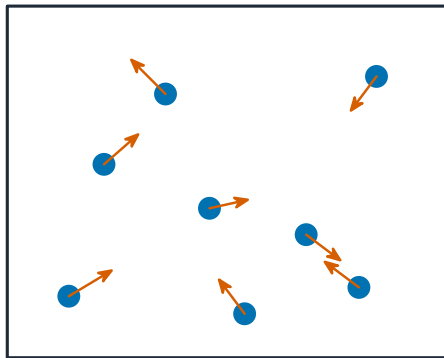
A-Level Physics

# Kinetic theory: the assumptions

- 1 many identical molecules in **random motion**
- 2 their own volume is negligible
- 3 collisions are brief; straight lines between
- 4 no **intermolecular** forces except in a collision
- 5 collisions are **elastic**
- 6 Newton's laws apply

Elastic is what keeps a gas from **cooling itself**. The model fails at **high**  $p$  (volume matters) and **low**  $T$  (forces matter).

# Assumptions

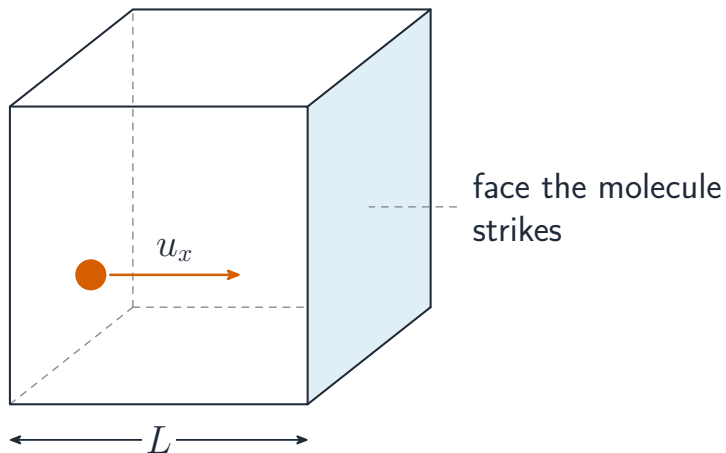


1. many identical molecules, in continuous **random motion**
2. their own volume is **tiny** next to the container
3. straight lines between hits - intermolecular forces **ignored**
4. collisions are **elastic**: no kinetic energy is lost

poor at very high pressure (volume matters) or very low temperature (forces matter)

The key assumptions of the kinetic theory of an ideal gas

# Where the pressure comes from



One molecule, one wall, in a cube of side  $L$ :

- bounce: the **momentum** change is  $\Delta p_x = -2mu_1$ , so the wall takes an **impulse** of  $+2mu_1$
- next hit after  $\Delta t = 2L/u_1$
- so  $F_1 = \frac{\Delta p}{\Delta t} = \frac{mu_1^2}{L}$

Sum over  $N$ , then use  $\langle u_x^2 \rangle = \frac{1}{3} \langle c^2 \rangle$ :

$$pV = \frac{1}{3} Nm \langle c^2 \rangle$$

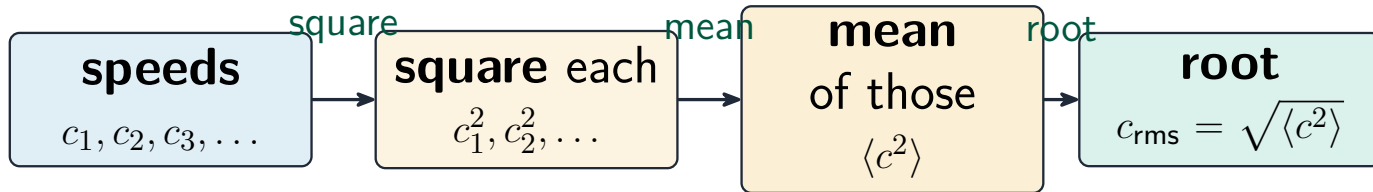
The  $\frac{1}{3}$  is **symmetry**: no direction is special, so each axis carries a third of the motion.

# Root-mean-square speed

**r.m.s. speed**  $= \sqrt{\langle c^2 \rangle}$ : square each speed, average, then square-root. It sits slightly **above** the mean speed – squaring weights the fast molecules more.

The gas needs  $\langle c^2 \rangle$ , not  $\langle c \rangle$ , because **energy** goes as  $c^2$ . Averaging the speeds first gives the wrong answer.

## Root-mean-square speed, in detail



read the name **backwards**: root of the **m**ean of the **s**quares -  
squaring weights fast molecules more, so  $c_{\text{rms}}$  is slightly above the mean speed

Square every speed, average, then square-root – in that order. Averaging the speeds first gives a different, smaller number, because the fast molecules count for more once squared.

## Exercises 2.1-2.3

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**2.1** State **two** assumptions of the kinetic theory of gases. [2]

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**2.2** Explain how gas molecules cause **pressure** on the walls of a container. [2]

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**2.3** State what is meant by the **root-mean-square (r.m.s.)** speed. [1]

## Exercises 2.1-2.3

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2.1 State **two** assumptions of the kinetic theory of gases. [2]

2.1

- any two: many identical molecules in **random motion**; molecular **volume negligible**; **no forces** except during collisions; collisions are **elastic**; collision time negligible

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2.2 Explain how gas molecules cause **pressure** on the walls of a container. [2]

2.2

- molecules collide with the walls and **change momentum** (rebound)
- this exerts a **force** on the wall; force over the wall area is the **pressure**

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2.3 State what is meant by the **root-mean-square (r.m.s.) speed**. [1]

2.3

- the **square root of the mean of the squared speeds** of the molecules ( $\sqrt{\langle c^2 \rangle}$ )

## Exercises 2.4·3.1

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2.4 Write the relationship  $pV = \frac{1}{3}Nm\langle c^2 \rangle$  in words.

[1]

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3.1 The mean translational kinetic energy of a gas molecule is:

[1]

☐

A  $kT$

B  $\frac{3}{2}kT$

C  $\frac{1}{2}mv^2$  only

D  $RT$

## Exercises 2.4·3.1

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2.4 Write the relationship  $pV = \frac{1}{3}Nm\langle c^2 \rangle$  in words. [1]

2.4

- the product of pressure and volume equals **one third** of (the number of molecules  $\times$  molecular mass  $\times$  mean-square speed)

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3.1 The **mean translational kinetic energy** of a gas molecule is: [1]

- ☐ **A**  $kT$   
**B**  $\frac{3}{2}kT$   
**C**  $\frac{1}{2}mv^2$  only  
**D**  $RT$

3.1 B

- mean kinetic energy of a molecule is  $\frac{3}{2}kT$
- A** is  $kT$ ; **C** is a speed formula, not the mean KE; **D** is  $RT$ , which is for a mole, not one molecule

## Exercises 3.2-3.3

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**3.2** Calculate the mean translational kinetic energy of a gas molecule at 300 K. ( $k = 1.38 \times 10^{-23}$ .) [2]

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**3.3** A gas at 300 K has molecules of mass  $5.3 \times 10^{-26}$  kg. Calculate the **r.m.s. speed**. ( $k = 1.38 \times 10^{-23}$ .) [3]

## Exercises 3.2·3.3

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**3.2** Calculate the mean translational kinetic energy of a gas molecule at 300 K. ( $k = 1.38 \times 10^{-23}$ .) [2]

**3.2**

- $E = \frac{3}{2}kT$

$$E = \frac{3}{2}kT = \frac{3}{2}(1.38 \times 10^{-23} \text{ J K}^{-1})(300 \text{ K}) = \mathbf{6.2 \times 10^{-21} \text{ J}}$$

**3.3** A gas at 300 K has molecules of mass  $5.3 \times 10^{-26}$  kg. Calculate the **r.m.s. speed**. ( $k = 1.38 \times 10^{-23}$ .) [3]

**3.3**

- $\frac{1}{2}m\langle c^2 \rangle = \frac{3}{2}kT \Rightarrow \langle c^2 \rangle = 3kT/m$

$$c_{\text{r.m.s.}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(1.38 \times 10^{-23} \text{ J K}^{-1})(300 \text{ K})}{5.3 \times 10^{-26} \text{ kg}}} = \mathbf{4.8 \times 10^2 \frac{\text{m}}{\text{s}}}$$

## Exercise 3.4

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**3.4** Explain, using the kinetic theory, why the pressure of a fixed mass of gas **increases** when it is heated at **constant volume**. [2]

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### 3.4

- heating raises the **mean KE / mean speed** of the molecules
- they hit the walls **harder and more often**, so the pressure rises (volume fixed)

## Exercise 3.5 (a)

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**3.5** A fixed sample of an ideal gas at thermodynamic temperature  $T$  has internal energy  $U$ .

(a) **State** what is meant by the **internal energy** of a system.

[2]

## Exercise 3.5 (a)

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**3.5** A fixed sample of an ideal gas at thermodynamic temperature  $T$  has internal energy  $U$ .

- (a) **State** what is meant by the **internal energy** of a system. [2]
- (a) the **sum of the random kinetic and potential energies** of all the molecules
- “random” means the motion has no overall direction – the whole container moving does not count

## Exercise 3.5 (b)

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**3.5** A fixed sample of an ideal gas at thermodynamic temperature  $T$  has internal energy  $U$ .

(b) **Explain** why the internal energy of an **ideal** gas is directly proportional to its thermodynamic temperature. [2]

## Exercise 3.5 (b)

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**3.5** A fixed sample of an ideal gas at thermodynamic temperature  $T$  has internal energy  $U$ .

(b) **Explain** why the internal energy of an **ideal** gas is directly proportional to its thermodynamic temperature. [2]

(b) in an ideal gas the molecules exert **no forces** on one another except during collisions, so the total **potential** energy is zero and the internal energy is entirely kinetic

- the mean kinetic energy of a molecule is  $\frac{3}{2}kT$ , i.e. proportional to  $T$ , so the total is proportional to  $T$  as well

## Exercise 3.5 (c)

**3.5** A fixed sample of an ideal gas at thermodynamic temperature  $T$  has internal energy  $U$ .

(c) The gas is first **compressed**, which raises its temperature to  $3T$ ; work  $W$  is done on it during this compression. It is then **cooled at constant volume** back to  $2T$ . **Complete** the table, giving each entry in terms of some or all of  $W$  and  $U$ . [4]

| process             | work done on the gas | thermal energy supplied to the gas | increase in internal energy |
|---------------------|----------------------|------------------------------------|-----------------------------|
| compression to $3T$ | $W$                  | _____                              | _____                       |
| cooling to $2T$     | _____                | _____                              | _____                       |

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| process             | work done on the gas | thermal energy supplied to the gas | increase in internal energy |
|---------------------|----------------------|------------------------------------|-----------------------------|
| compression to $3T$ | $W$                  | _____                              | _____                       |
| cooling to $2T$     | _____                | _____                              | _____                       |

## Full solution 3.5 (c)

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(c) internal energy is proportional to  $T$ , so at  $T$  it is  $U$ , at  $3T$  it is  $3U$  and at  $2T$  it is  $2U$

| process                | work done on<br>the gas | thermal energy<br>supplied | increase in internal<br>energy |
|------------------------|-------------------------|----------------------------|--------------------------------|
| compression to<br>$3T$ | $W$                     | $2U - W$                   | $+2U$                          |
| cooling to $2T$        | $0$                     | $-U$                       | $-U$                           |

- $+2U$ ;  $-U$ ; the zero work; both thermal-energy entries, from  $\Delta U = q + w$

## Exercise 3.5 (d)

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**3.5** A fixed sample of an ideal gas at thermodynamic temperature  $T$  has internal energy  $U$ .

(d) **Explain** why the work done on the gas during the second process is what it is. [1]

## Exercise 3.5 (d)

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**3.5** A fixed sample of an ideal gas at thermodynamic temperature  $T$  has internal energy  $U$ .

(d) **Explain** why the work done on the gas during the second process is what it is. [1]

(d) at **constant volume** the gas neither expands nor is compressed, so no work is done on or by it