

Exercise walk-through

15.3.1

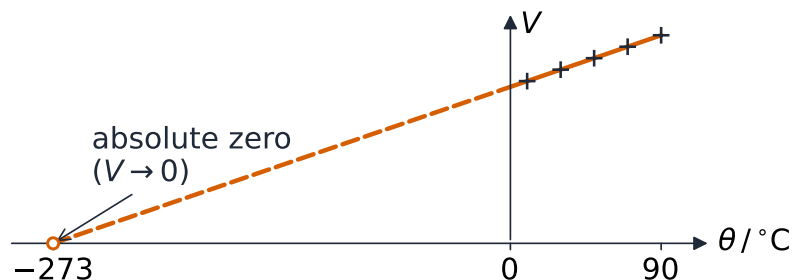
**Deriving $pV = \text{one third } Nm$
mean-square speed, and what it
says about temperature**

A-Level Physics

Charles's law and absolute zero

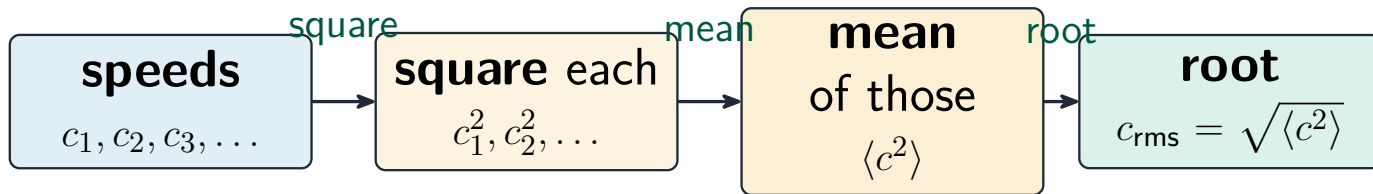
At constant pressure the volume climbs **linearly** with temperature. Extrapolate the line back to **zero volume** and it meets $\approx -273^{\circ}\text{C}$.

Every gas points at the **same** intercept – which is what makes absolute zero a property of **temperature**, not of any one gas.



$V \propto T$ · Charles: extrapolates to absolute zero

Root-mean-square speed, in detail



read the name **backwards**: root of the **m**ean of the **s**quares -
squaring weights fast molecules more, so c_{rms} is slightly above the mean speed

Square every speed, average, then square-root – in that order. Averaging the speeds first gives a different, smaller number, because the fast molecules count for more once squared.

Average translational KE

The **average translational kinetic energy**, from two expressions for the same pV – the second using the **mean square** speed $\langle c^2 \rangle$:

$$pV = NkT, \quad pV = \frac{1}{3}Nm\langle c^2 \rangle$$

Set equal, cancel N , multiply by $\frac{3}{2}$:

$$\langle E_k \rangle = \frac{1}{2}m\langle c^2 \rangle = \frac{3}{2}kT$$

The average molecular KE depends on **temperature alone** – not the gas, not the pressure. Squeezing a gas at fixed T gives more wall hits, **not** faster molecules.

Exercises 2.1-2.2

2.1 State **three** assumptions of the kinetic theory of gases.

[3]

2.2 Explain why the momentum change of a molecule rebounding from a wall is $2mv_x$ and not mv_x .

[2]

Exercises 2.1·2.2

2.1 State **three** assumptions of the kinetic theory of gases.

[3]

2.1

- any **three** of: a large number of molecules in **random** motion; the **volume of the molecules** is negligible compared with the volume of the container; collisions are **perfectly elastic**; the duration of a collision is negligible compared with the time between collisions; there are **no forces** between molecules except during collisions

2.2 Explain why the momentum change of a molecule rebounding from a wall is $2mv_x$ and not mv_x .

[2]

2.2

- momentum is a **vector**: the molecule arrives with $+mv_x$ and leaves with $-mv_x$
- the **change** is $(-mv_x) - (+mv_x) = -2mv_x$, so its magnitude is $2mv_x$

Exercises 2.3-2.4

2.3 Show that the time between successive collisions of one molecule with the **same** wall of a cubical box of side L is $\frac{2L}{v_x}$. [2]

2.4 State what is meant by the **mean-square speed** and by the **root-mean-square speed**. [2]

Exercises 2.3·2.4

2.3 Show that the time between successive collisions of one molecule with the **same** wall of a cubical box of side L is $\frac{2L}{v_x}$. [2]

2.3

- after rebounding, the molecule must cross the box and return before it can hit the same wall again, a distance of $2L$
- its component of velocity in that direction is v_x and is unchanged in magnitude, so

$$\Delta t = \frac{\text{distance}}{\text{speed}} = \frac{2L}{v_x}$$

2.4 State what is meant by the **mean-square speed** and by the **root-mean-square speed**. [2]

$$\overline{v^2} = \text{mean-square speed} = \overline{v_x^2 + v_y^2 + v_z^2}$$

2.4

- the **mean-square speed** $\langle v^2 \rangle$ is the average of the squares of the speeds of all the molecules
- the **root-mean-square speed** is the square root of that, $c_{\text{r.m.s.}} = \sqrt{\langle v^2 \rangle}$

Exercises 2.5·3.1

2.5 Explain why $\langle v_x^2 \rangle = \frac{1}{3} \langle c^2 \rangle$.

[2]

3.1 The thermodynamic temperature of an ideal gas is doubled. The root-mean-square speed of its molecules is multiplied by

[1]

☐

A $\sqrt{2}$

B 2

C 4

D $\frac{1}{\sqrt{2}}$

Exercises 2.5·3.1

2.5 Explain why $\langle v_x^2 \rangle = \frac{1}{3} \langle c^2 \rangle$.

[2]

2.5

- the motion is **random**, so no direction is preferred and $\langle v_x^2 \rangle = \langle v_y^2 \rangle = \langle v_z^2 \rangle$
- since $c^2 = v_x^2 + v_y^2 + v_z^2$, taking means gives $\langle c^2 \rangle = 3\langle v_x^2 \rangle$, so $\langle v_x^2 \rangle = \frac{1}{3} \langle c^2 \rangle$

3.1 The thermodynamic temperature of an ideal gas is doubled. The root-mean-square speed of its molecules is multiplied by

[1]

- ☐ A $\sqrt{2}$
☐ B 2
☐ C 4
☐ D $\frac{1}{\sqrt{2}}$

3.1 A

- $c_{\text{r.m.s.}} = \sqrt{\frac{3kT}{m}}$, so $c_{\text{r.m.s.}} \propto \sqrt{T}$
- doubling T multiplies the speed by $\sqrt{2}$. **B** is the standard error – it is the mean *kinetic energy* that doubles, not the speed

Exercise 3.2 (a)

3.2 The kinetic theory gives $pV = \frac{1}{3}Nm\langle c^2 \rangle$, and the ideal-gas equation gives $pV = NkT$.

(a) Use these to show that the mean translational kinetic energy of a molecule is $\frac{3}{2}kT$. [3]

Exercise 3.2 (a)

3.2 The kinetic theory gives $pV = \frac{1}{3}Nm\langle c^2 \rangle$, and the ideal-gas equation gives $pV = NkT$.

(a) Use these to show that the mean translational kinetic energy of a molecule is $\frac{3}{2}kT$. [3]

(a)

- both expressions equal pV , so set them equal

$$\frac{1}{3}Nm\langle c^2 \rangle = NkT$$

- the N cancels

$$\frac{1}{3}m\langle c^2 \rangle = kT$$

- multiply both sides by $\frac{3}{2}$

$$\frac{1}{2}m\langle c^2 \rangle = \frac{3}{2}kT$$

Exercise 3.2 (b)

3.2 The kinetic theory gives $pV = \frac{1}{3}Nm\langle c^2 \rangle$, and the ideal-gas equation gives $pV = NkT$.

(b) Calculate the mean translational kinetic energy of a molecule at 27 °C. Take $k = 1.38 \times 10^{-23} \text{ J K}^{-1}$. [2]

Exercise 3.2 (b)

3.2 The kinetic theory gives $pV = \frac{1}{3}Nm\langle c^2 \rangle$, and the ideal-gas equation gives $pV = NkT$.

(b) Calculate the mean translational kinetic energy of a molecule at 27 °C. Take $k = 1.38 \times 10^{-23} \text{ J K}^{-1}$. [2]

(b)

- convert the temperature to kelvin, $T = 300 \text{ K}$

$$E = \frac{3}{2}kT = 1.5 \times 1.38 \times 10^{-23} \text{ J K}^{-1} \times 300 \text{ K} = \mathbf{6.2 \times 10^{-21} \text{ J}}$$

Exercise 3.2 (c)

3.2 The kinetic theory gives $pV = \frac{1}{3}Nm\langle c^2 \rangle$, and the ideal-gas equation gives $pV = NkT$.

(c) State what this result shows about the meaning of thermodynamic temperature. [1]

Exercise 3.2 (c)

3.2 The kinetic theory gives $pV = \frac{1}{3}Nm\langle c^2 \rangle$, and the ideal-gas equation gives $pV = NkT$.

(c) State what this result shows about the meaning of thermodynamic temperature. [1]

(c)

- thermodynamic temperature is a measure of the **mean translational kinetic energy** of the molecules, and it is the same for every ideal gas at that temperature

Exercise 3.3 (a)

3.3 Helium, of molar mass 4.0 g mol^{-1} , and oxygen, of molar mass 32 g mol^{-1} , are both held at 300 K . Take $N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$ and $k = 1.38 \times 10^{-23} \text{ J K}^{-1}$.

(a) Calculate the mass of one helium atom.

[2]

Exercise 3.3 (a)

3.3 Helium, of molar mass 4.0 g mol^{-1} , and oxygen, of molar mass 32 g mol^{-1} , are both held at 300 K . Take $N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$ and $k = 1.38 \times 10^{-23} \text{ J K}^{-1}$.

(a) Calculate the mass of one helium atom. [2]

(a)

- one mole contains N_A atoms

$$m = \frac{M_m}{N_A} = \frac{4.0 \times 10^{-3} \text{ kg mol}^{-1}}{6.02 \times 10^{23} \text{ mol}^{-1}} = \mathbf{6.6 \times 10^{-27} \text{ kg}}$$

Exercise 3.3 (b)

3.3 Helium, of molar mass 4.0 g mol^{-1} , and oxygen, of molar mass 32 g mol^{-1} , are both held at 300 K . Take $N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$ and $k = 1.38 \times 10^{-23} \text{ J K}^{-1}$.

(b) Calculate the root-mean-square speed of a helium atom at 300 K . [3]

Exercise 3.3 (b)

3.3 Helium, of molar mass 4.0 g mol^{-1} , and oxygen, of molar mass 32 g mol^{-1} , are both held at 300 K . Take $N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$ and $k = 1.38 \times 10^{-23} \text{ J K}^{-1}$.

(b) Calculate the root-mean-square speed of a helium atom at 300 K . [3]

(b)

- rearrange $\frac{1}{2}m\langle c^2 \rangle = \frac{3}{2}kT$

$$\langle c^2 \rangle = \frac{3kT}{m} = \frac{3 \times 1.38 \times 10^{-23} \text{ J K}^{-1} \times 300 \text{ K}}{6.6 \times 10^{-27} \text{ kg}} = 1.87 \times 10^6 \text{ m}^2 \text{ s}^{-2}$$

$$c_{\text{r.m.s.}} = \sqrt{\langle c^2 \rangle} = \sqrt{1.87 \times 10^6 \text{ m}^2 \text{ s}^{-2}} = \mathbf{1.4 \times 10^3 \frac{m}{s}}$$

Exercise 3.3 (c)

3.3 Helium, of molar mass 4.0 g mol^{-1} , and oxygen, of molar mass 32 g mol^{-1} , are both held at 300 K . Take $N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$ and $k = 1.38 \times 10^{-23} \text{ J K}^{-1}$.

(c) Without calculating it exactly, state and explain how the root-mean-square speed of an oxygen molecule at the same temperature compares with that of a helium atom. [3]

Exercise 3.3 (c)

3.3 Helium, of molar mass 4.0 g mol^{-1} , and oxygen, of molar mass 32 g mol^{-1} , are both held at 300 K . Take $N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$ and $k = 1.38 \times 10^{-23} \text{ J K}^{-1}$.

(c) Without calculating it exactly, state and explain how the root-mean-square speed of an oxygen molecule at the same temperature compares with that of a helium atom. [3]

(c)

- at the same temperature both gases have the **same mean translational kinetic energy**, $\frac{3}{2}kT$
- so $\frac{1}{2}m\langle c^2 \rangle$ is the same for both, and $c_{\text{r.m.s.}} \propto \frac{1}{\sqrt{m}}$
- an oxygen molecule has **eight times** the mass, so its r.m.s. speed is $\frac{1}{\sqrt{8}}$ of the helium value – about **2.8 times smaller**, roughly $480 \frac{\text{m}}{\text{s}}$

Exercise 3.3 (d)

3.3 Helium, of molar mass 4.0 g mol^{-1} , and oxygen, of molar mass 32 g mol^{-1} , are both held at 300 K . Take $N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$ and $k = 1.38 \times 10^{-23} \text{ J K}^{-1}$.

(d) The helium is now heated to 600 K . State and explain the effect on the root-mean-square speed of its atoms. [2]

Exercise 3.3 (d)

3.3 Helium, of molar mass 4.0 g mol^{-1} , and oxygen, of molar mass 32 g mol^{-1} , are both held at 300 K . Take $N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$ and $k = 1.38 \times 10^{-23} \text{ J K}^{-1}$.

(d) The helium is now heated to 600 K . State and explain the effect on the root-mean-square speed of its atoms. [2]

(d)

- the r.m.s. speed **increases by a factor of $\sqrt{2}$** , to about $1.9 \times 10^3 \frac{\text{m}}{\text{s}}$
- because $c_{\text{r.m.s.}} \propto \sqrt{T}$, and the thermodynamic temperature has doubled

Exercise 3.3 (e)

3.3 Helium, of molar mass 4.0 g mol^{-1} , and oxygen, of molar mass 32 g mol^{-1} , are both held at 300 K . Take $N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$ and $k = 1.38 \times 10^{-23} \text{ J K}^{-1}$.

(e) Use the kinetic theory to explain why the pressure of a fixed mass of gas increases when it is heated at constant volume. [3]

Exercise 3.3 (e)

3.3 Helium, of molar mass 4.0 g mol^{-1} , and oxygen, of molar mass 32 g mol^{-1} , are both held at 300 K . Take $N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$ and $k = 1.38 \times 10^{-23} \text{ J K}^{-1}$.

(e) Use the kinetic theory to explain why the pressure of a fixed mass of gas increases when it is heated at constant volume. [3]

(e)

- heating raises the mean translational kinetic energy, so the molecules move **faster** on average
- they therefore hit the walls **more frequently**
- and each collision produces a **greater change of momentum**; both effects raise the average force on the walls, and with the area unchanged the pressure rises