

Exercise walk-through

15.2

Equation of state

A-Level Physics

The equation of state

An **ideal gas** obeys $pV \propto T$ exactly:

$$pV = nRT \quad \text{or} \quad pV = NkT$$

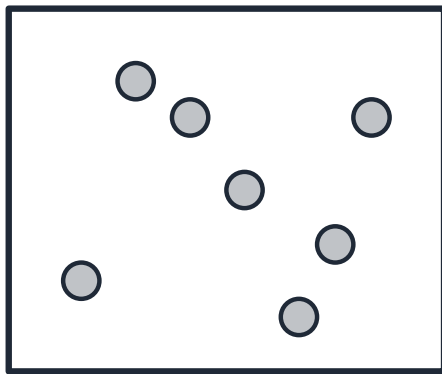
- the **molar gas constant**
- the **Boltzmann constant**
- $N = nN_A$, so $k = R/N_A$

$$R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1} - \text{per mole}$$

$$k = 1.38 \times 10^{-23} \text{ J K}^{-1} - \text{per molecule}$$

Match the form to your **amount**: moles $\rightarrow nRT$, molecules $\rightarrow NkT$. SI throughout – Pa, m^3 , and **kelvin**.

The equation of state, in detail



gas: p , V , T , n moles

$$pV = nRT$$

Every quantity in $pV = nRT$ is in SI units: pressure in Pa, volume in m^3 , and temperature in **kelvin** – never $^{\circ}\text{C}$.

Worked example – how many moles?

A 0.020 m^3 cylinder holds gas at 27°C and $2.0 \times 10^5 \text{ Pa}$. Find n . ($R = 8.31$)

First to kelvin: $T = 27 + 273 = 300 \text{ K}$.

$$n = \frac{pV}{RT} = \frac{(2.0 \times 10^5)(0.020)}{8.31 \times 300} \approx 1.6 \text{ mol}$$

The kelvin conversion is the **first** line, every time. 27 in place of 300 would inflate n by a factor of **11**.

Exercises 2.1-2.3

2.1 State what is meant by an **ideal gas**. [1]

2.2 Write the **two** forms of the ideal-gas equation of state. [2]

2.3 State the value of the **molar gas constant** R . [1]

Exercises 2.1-2.3

2.1 State what is meant by an **ideal gas**.

[1]

2.1

- a gas that obeys $pV \propto T$ (i.e. $pV = nRT$) exactly, at all p and T

2.2 Write the **two** forms of the ideal-gas equation of state.

[2]

2.2

- $pV = nRT$
- $pV = NkT$

2.3 State the value of the **molar gas constant** R .

[1]

2.3

- $R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$

Exercises 2.4·3.1

2.4 A graph of pV against T for a fixed mass of ideal gas is a straight line through the origin. State what its **gradient** is equal to. [1]

3.1 In $pV = nRT$, the temperature T must be in: [1]

☐

A °C

B K

C J

D mol

Exercises 2.4·3.1

2.4 A graph of pV against T for a fixed mass of ideal gas is a straight line through the origin. State what its **gradient** is equal to. [1]

2.4

- Nk (the number of molecules $\times$ the Boltzmann constant)

3.1 In $pV = nRT$, the temperature T must be in: [1]

☐

A °C

B K

C J

D mol

3.1 B

- T in $pV = nRT$ must be in **kelvin**
- **A** is Celsius; **C** is energy; **D** is amount of substance

Exercise 3.2

3.2 0.50 mol of an ideal gas at 300 K occupies 0.020 m³. Calculate the **pressure**.
($R = 8.31$.) [3]

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3.2

- $p = nRT/V$

$$p = \frac{nRT}{V} = \frac{(0.50 \text{ mol})(8.31 \text{ J mol}^{-1} \text{ K}^{-1})(300 \text{ K})}{0.020 \text{ m}^3} = \mathbf{6.2 \times 10^4 \text{ Pa}}$$

Exercise 3.3 (a)

3.3 For a fixed mass of gas, the graph of pV against T is a straight line through the origin passing through (300 K, 600 J).

(a) State what this tells you about the gas.

[1]

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(a) State what this tells you about the gas. [1]

(a) it is an **ideal gas**: $pV \propto T$ (the line is straight and passes through the origin)

Exercise 3.3 (b)

3.3 For a fixed mass of gas, the graph of pV against T is a straight line through the origin passing through (300 K, 600 J).

(b) Determine the number of molecules N in the sample. ($k = 1.38 \times 10^{-23}$.) [3]

Exercise 3.3 (b)

3.3 For a fixed mass of gas, the graph of pV against T is a straight line through the origin passing through (300 K, 600 J).

(b) Determine the number of molecules N in the sample. ($k = 1.38 \times 10^{-23}$.) [3]

(b) gradient = $600 \text{ J} / 300 \text{ K} = 2.0 \text{ J K}^{-1}$

- this equals Nk

$$N = \frac{\text{gradient}}{k} = \frac{2.0 \text{ J K}^{-1}}{1.38 \times 10^{-23} \text{ J K}^{-1}} = 1.4 \times 10^{23}$$

Exercise 3.4

3.4 State Boyle's law.

[1]

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[1]

3.4

- at constant temperature (and amount of gas), the pressure is **inversely proportional** to the volume ($pV = \text{constant}$)

Exercise 3.5 (a)

3.5 For an ideal gas, two equations are used together:

$$pV = NkT \quad \text{and} \quad pV = \frac{1}{3}Nm\langle c^2 \rangle$$

(a) **State** the meaning of the symbol T in the first equation, and **identify** the constant k . [2]

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(a) **State** the meaning of the symbol T in the first equation, and **identify** the constant k . [2]

(a) T is the **thermodynamic (absolute) temperature**, measured in kelvin

- k is the **Boltzmann constant**

Exercise 3.5 (b)

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(b) **State** the meanings of the symbols m and $\langle c^2 \rangle$ in the second equation. [2]

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(b) **State** the meanings of the symbols m and $\langle c^2 \rangle$ in the second equation. [2]

(b) m is the mass of **one molecule**

- $\langle c^2 \rangle$ is the **mean square speed** – the average of the squares of the molecular speeds, not the square of the average

Exercise 3.5 (c)

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$$pV = NkT \quad \text{and} \quad pV = \frac{1}{3}Nm\langle c^2 \rangle$$

(c) Use the two equations to **derive** an expression, in terms of k and T only, for the mean kinetic energy E_K of one molecule of the gas. [3]

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(c) Use the two equations to **derive** an expression, in terms of k and T only, for the mean kinetic energy E_K of one molecule of the gas. [3]

(c) set the two right-hand sides equal: $NkT = \frac{1}{3}Nm\langle c^2 \rangle$

- so $\frac{1}{3}m\langle c^2 \rangle = kT$
- the mean kinetic energy of one molecule is $E_K = \frac{1}{2}m\langle c^2 \rangle$
- which is $\frac{3}{2}$ times the previous line, so $E_K = \frac{3}{2}kT$

Exercise 3.5 (d)

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$$pV = NkT \quad \text{and} \quad pV = \frac{1}{3}Nm\langle c^2 \rangle$$

(d) Hence **state and explain** how the root-mean-square speed of the molecules changes when the thermodynamic temperature of the gas is **quadrupled**. [2]

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(d) Hence **state and explain** how the root-mean-square speed of the molecules changes when the thermodynamic temperature of the gas is **quadrupled**. [2]

(d) $\frac{1}{2}m\langle c^2 \rangle \propto T$, so $\langle c^2 \rangle \propto T$ and $c_{\text{r.m.s.}} \propto \sqrt{T}$

- quadrupling T therefore **doubles** the r.m.s. speed