

Exercise walk-through

15.1

The mole

A-Level Physics

The mole

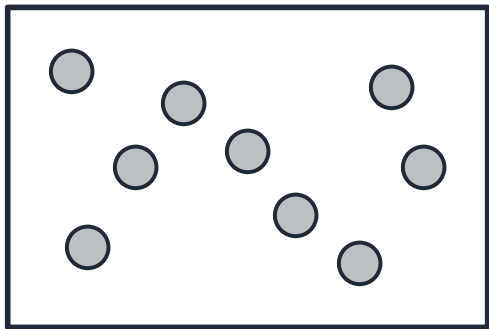
Amount of substance n is measured in **moles**. One mole holds **Avogadro constant** particles
 – **atoms** for a **monatomic** gas like helium, molecules for O_2 :

$$N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$$

Number of particles $N = nN_A$. Molar mass in grams is the relative formula mass – 18 g of water is one mole.

Count particles with moles, not with 10^{23} on every line. n is the quantity the gas laws actually use.

One mole, counted



1 mole

$$= 6.02 \times 10^{23}$$

particles (N_A)

A mole is 6.02×10^{23} particles – the number chosen so that a mole of a substance weighs its relative molecular mass in grams.

One mole is 6.02×10^{23} particles – whatever the particle is. The molar mass in grams equals the relative formula mass, so 18 g of water is exactly one mole.

Exercises 2.1-2.3

2.1 State the SI **unit** of amount of substance. [1]

2.2 State the value of the **Avogadro constant**. [1]

2.3 How many molecules are there in 2.0 mol of a gas? [2]

Exercises 2.1-2.3

2.1 State the SI **unit** of amount of substance.

[1]

2.1

- the **mole** (mol)

2.2 State the value of the **Avogadro constant**.

[1]

2.2

- $N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$

2.3 How many molecules are there in 2.0 mol of a gas?

[2]

2.3

- $N = nN_A$

$$N = nN_A = (2.0 \text{ mol})(6.02 \times 10^{23} \text{ mol}^{-1}) = \mathbf{1.2 \times 10^{24}}$$

Exercises 3.1·3.2

3.1 One mole of oxygen gas (O_2) contains:

[1]

☐

A 6.02×10^{23} atoms

B 6.02×10^{23} molecules

C 32 molecules

D 16 molecules

3.2 A gas sample contains 3.0×10^{24} molecules. Find the number of **moles**.

[2]

Exercises 3.1·3.2

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3.1 B

- one mole contains 6.02×10^{23} molecules
- **A** is $N_A/10$; **C** is $2N_A$; **D** is a mass, not a number

3.2 A gas sample contains 3.0×10^{24} molecules. Find the number of **moles**.

[2]

3.2

- $n = N/N_A$

$$n = \frac{N}{N_A} = \frac{3.0 \times 10^{24}}{6.02 \times 10^{23} \text{ mol}^{-1}} = \mathbf{5.0 \text{ mol}}$$

Exercise 3.3

3.3 The molar mass of water is 18 g mol^{-1} . Calculate the mass of **one** water molecule. [2]

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3.3

- $m_0 = M_{\text{m}}/N_{\text{A}}$

$$m_0 = \frac{M_{\text{m}}}{N_{\text{A}}} = \frac{0.018 \text{ kg mol}^{-1}}{6.02 \times 10^{23} \text{ mol}^{-1}} = \mathbf{3.0 \times 10^{-26} \text{ kg}}$$

Exercise 3.4 (a)

3.4 A sealed cylinder holds an ideal gas. The Avogadro constant is $6.02 \times 10^{23} \text{ mol}^{-1}$, the molar gas constant is $8.31 \text{ J mol}^{-1} \text{ K}^{-1}$ and the Boltzmann constant is $1.38 \times 10^{-23} \text{ J K}^{-1}$.

(a) **State** what is meant by the **Avogadro constant**.

[1]

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(a) **State** what is meant by the **Avogadro constant**. [1]

(a) the number of particles per unit amount of substance – the number of particles in one mole

Exercise 3.4 (b)

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(b) **State** the relationship between the Avogadro constant N_A , the molar gas constant R and the Boltzmann constant k . [1]

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(b) **State** the relationship between the Avogadro constant N_A , the molar gas constant R and the Boltzmann constant k . [1]

(b) $N_A = R/k$

Exercise 3.4 (c)

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(c) The cylinder holds 0.32 mol of the gas. **Determine** the number of molecules in it.[2]

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(c) $N = nN_A$

$$N = nN_A = (0.32 \text{ mol})(6.02 \times 10^{23} \text{ mol}^{-1}) = \mathbf{1.9 \times 10^{23}}$$

Setup for exercise 3.4

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Exercise 3.4 (d)

(d) A second, identical cylinder holds **twice** as many molecules of the same gas at the same temperature T . **Complete** the table, giving each entry in terms of some or all of N , m , T , V , k and N_A , where sample X contains N molecules each of mass m in volume V . [4]

quantity	sample X	sample Y
pressure	_____	_____
amount of substance	_____	_____
mean kinetic energy per molecule	_____	_____
internal energy	_____	_____

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internal energy	_____	_____

(d) sample Y has $2N$ molecules in the same volume at the same temperature

quantity	sample X	sample Y
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Full solution 3.4 (d)

(d) sample Y has $2N$ molecules in the same volume at the same temperature

quantity	sample X	sample Y
pressure	NkT/V	$2NkT/V$
amount of substance	N/N_A	$2N/N_A$
mean kinetic energy per molecule	$\frac{3}{2}kT$	$\frac{3}{2}kT$
internal energy	$\frac{3}{2}NkT$	$3NkT$

- pressures; amounts; both mean kinetic energies the same; internal energies

Exercise 3.4 (e)

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(e) **Explain** why the mean kinetic energy per molecule is the same in both samples, while the internal energies differ. [2]

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(e) the mean kinetic energy of a molecule is $\frac{3}{2}kT$, which depends **only on the temperature** – and both samples are at T

- internal energy is the **total** for all the molecules, so twice as many molecules at the same energy each gives twice the internal energy

Exercise 3.4 (f)

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(f) The temperature of sample X is now varied. In the space below, **sketch** how the root-mean-square speed of its molecules varies with thermodynamic temperature. **Label** both axes. [2]

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(f) axes labelled *root-mean-square speed* and *thermodynamic temperature / K*

- a curve from the **origin** rising with a **decreasing** gradient, since $c_{\text{r.m.s.}} \propto \sqrt{T}$ – quadrupling T only doubles the speed