

Exercise walk-through

13.4

Gravitational potential

A-Level Physics

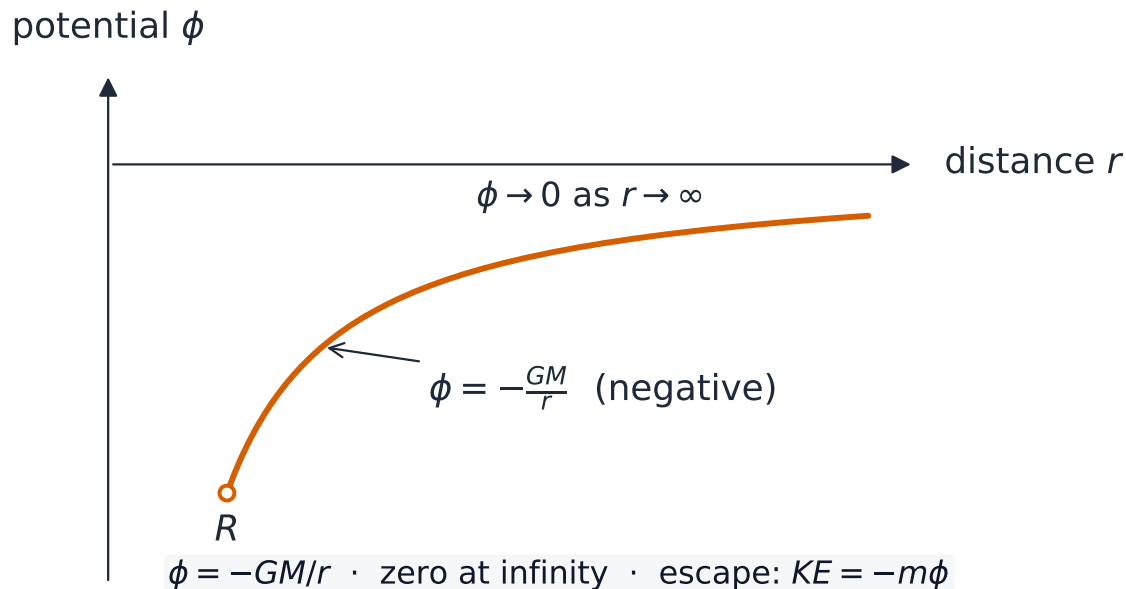
Gravitational potential

Potential infinity : is the work done **per unit mass** bringing a test mass in from

$$\phi = \frac{W}{m} = -\frac{GM}{r}, \quad [\phi] = \text{J kg}^{-1}$$

Zero at infinity, so ϕ is **negative** everywhere else – gravity did the work for you on the way in. ϕ is a **scalar**: just add them.

Gravitational potential, in detail



The gravitational potential well: $\phi = -GM/r$ is negative, rising to zero at infinity

Link with $\Delta E_p = mg\Delta h$

For small height changes near the surface, r barely changes, so $\Delta E_p \approx mg\Delta h$.

For large changes (a satellite moving to a higher orbit) use $-GMm/r$ at each radius and take the difference:

Potential energy and escape

A mass m sitting where the potential is ϕ :

$$E_p = m\phi = -\frac{GMm}{r}$$

Raising a satellite from r_1 to r_2 :

$$\Delta E_p = GMm \left(\frac{1}{r_1} - \frac{1}{r_2} \right) > 0$$

To escape, the kinetic energy must match the well's depth:

$$\frac{1}{2}mv_{\text{esc}}^2 = \frac{GMm}{r} \Rightarrow v_{\text{esc}} = \sqrt{\frac{2GM}{r}}$$

$mg\Delta h$ is only the **small-height** shortcut, where r hardly changes. Over orbital distances you must difference $-GMm/r$ at each radius.

Escape needs **no rocket left over**: reach v_{esc} and you coast to infinity, arriving with zero speed.

The energy of two point masses

Potential energy

The **gravitational potential energy** of two point masses is $E_p = -\frac{GMm}{r}$ – negative, because the zero is placed at infinity.

The constant

$G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$ is the **universal gravitational constant** – the same everywhere, which is what “universal” asserts.

Escape speed

Escape needs $\frac{1}{2}mv^2 \geq \frac{GMm}{r}$, so $v_{\text{esc}} = \sqrt{2GM/r}$ – independent of the escaping mass.

The minus sign is not decoration: it says work must be *done* to separate the masses, and it is what makes a bound orbit have negative total energy.

Exercises 2.1-2.3

2.1 Define **gravitational potential**.

[1]

2.2 State **why** gravitational potential is **negative**.

[1]

2.3 State the unit of gravitational potential.

[1]

Exercises 2.1-2.3

2.1 Define **gravitational potential**.

[1]

2.1

- the **work done per unit mass** in bringing a small test mass from **infinity** to that point

2.2 State **why** gravitational potential is **negative**.

[1]

2.2

- gravity is **attractive**, so it does the work as the mass moves in from infinity (where $\phi = 0$); the potential therefore becomes **negative**

2.3 State the unit of gravitational potential.

[1]

2.3

- J kg^{-1}

Exercises 2.4·3.1

2.4 Write the equation for the **gravitational potential energy** of two point masses.[1]

3.1 The gravitational potential at **infinity** is: [1]

☐

A a maximum positive value

B zero

C a large negative value

D infinite

Exercises 2.4·3.1

2.4 Write the equation for the **gravitational potential energy** of two point masses.[1]

2.4

- $E_P = -GMm/r$

3.1 The gravitational potential at **infinity** is: [1]

☐

A a maximum positive value

B zero

C a large negative value

D infinite

3.1 B

- gravitational potential is defined as **zero** at infinity
- **A** would be a surface value; **C** is a typical Earth-surface ϕ ; **D** is not the definition

Exercises 3.2·3.3

3.2 Calculate the gravitational potential at the Earth's surface. ($M = 6.0 \times 10^{24}$ kg, $R = 6.4 \times 10^6$ m, $G = 6.67 \times 10^{-11}$.) [3]

3.3 Calculate the gravitational potential energy of a 1200 kg satellite at $r = 7.0 \times 10^6$ m from the Earth's centre. [2]

Exercises 3.2-3.3

3.2 Calculate the gravitational potential at the Earth's surface. ($M = 6.0 \times 10^{24}$ kg, $R = 6.4 \times 10^6$ m, $G = 6.67 \times 10^{-11}$.) [3]

3.2

- $\phi = -GM/R$

$$\phi = -\frac{GM}{R} = -\frac{(6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2})(6.0 \times 10^{24} \text{ kg})}{6.4 \times 10^6 \text{ m}} = -\mathbf{6.3 \times 10^7 \text{ J kg}^{-1}}$$

3.3 Calculate the gravitational potential energy of a 1200 kg satellite at $r = 7.0 \times 10^6$ m from the Earth's centre. [2]

3.3

- $E_P = -GMm/r$

$$E_P = -\frac{GMm}{r} = -\frac{(6.67 \times 10^{-11})(6.0 \times 10^{24})(1200)}{7.0 \times 10^6} = -\mathbf{6.9 \times 10^{10} \text{ J}}$$

Exercises 3.4-3.5

3.4 Explain why **two closer** masses have **more negative** gravitational potential energy. [2]

3.5 A 1000 kg satellite orbiting a planet is moved from an orbit of radius 7.0×10^6 m to one of radius 1.4×10^7 m. Its gravitational potential energy **increases** by 2.86×10^{10} J. Calculate the **mass of the planet**, and give a **unit**. ($G = 6.67 \times 10^{-11}$.) [4]

Exercises 3.4·3.5

3.4 Explain why **two closer** masses have **more negative** gravitational potential energy. [2]

3.4

- as r decreases, $-GMm/r$ becomes **more negative**
- the masses are more **tightly bound** – more work would be needed to separate them to infinity

3.5 A 1000 kg satellite orbiting a planet is moved from an orbit of radius 7.0×10^6 m to one of radius 1.4×10^7 m. Its gravitational potential energy **increases** by 2.86×10^{10} J. Calculate the **mass of the planet**, and give a **unit**. ($G = 6.67 \times 10^{-11}$.) [4]

3.5

- $\Delta E_P = GMm(1/r_1 - 1/r_2)$

$$\frac{1}{r_1} - \frac{1}{r_2} = \frac{1}{7.0 \times 10^6 \text{ m}} - \frac{1}{1.4 \times 10^7 \text{ m}} = 7.14 \times 10^{-8} \text{ m}^{-1}$$

$$M = \frac{\Delta E_P}{Gm(1/r_1 - 1/r_2)} = \frac{2.86 \times 10^{10}}{(6.67 \times 10^{-11})(1000)(7.14 \times 10^{-8})} = \mathbf{6.0 \times 10^{24} \text{ kg}}$$