

Exercise walk-through

13.2

Gravitational force between point masses

A-Level Physics

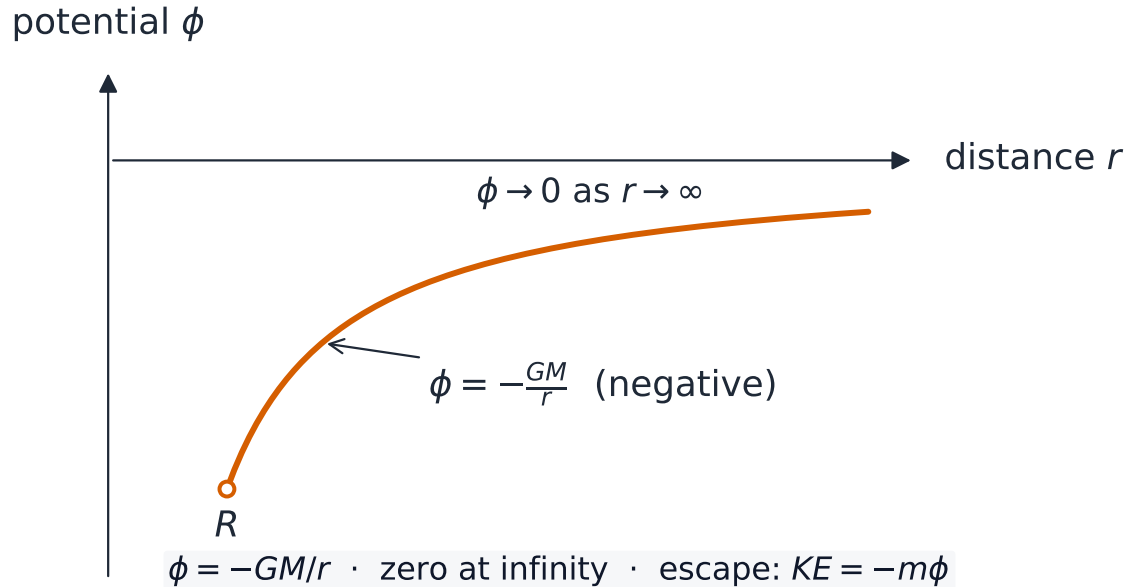
Gravitational potential

Potential infinity : is the work done **per unit mass** bringing a test mass in from

$$\phi = \frac{W}{m} = -\frac{GM}{r}, \quad [\phi] = \text{J kg}^{-1}$$

Zero at infinity, so ϕ is **negative** everywhere else – gravity did the work for you on the way in. ϕ is a **scalar**: just add them.

Gravitational potential, in detail



The gravitational potential well: $\phi = -GM/r$ is negative, rising to zero at infinity

Link with $\Delta E_p = mg\Delta h$

For small height changes near the surface, r barely changes, so $\Delta E_p \approx mg\Delta h$.

For large changes (a satellite moving to a higher orbit) use $-GMm/r$ at each radius and take the difference:

Potential energy and escape

A mass m sitting where the potential is ϕ :

$$E_p = m\phi = -\frac{GMm}{r}$$

Raising a satellite from r_1 to r_2 :

$$\Delta E_p = GMm \left(\frac{1}{r_1} - \frac{1}{r_2} \right) > 0$$

To escape, the kinetic energy must match the well's depth:

$$\frac{1}{2}mv_{\text{esc}}^2 = \frac{GMm}{r} \Rightarrow v_{\text{esc}} = \sqrt{\frac{2GM}{r}}$$

$mg\Delta h$ is only the **small-height** shortcut, where r hardly changes. Over orbital distances you must difference $-GMm/r$ at each radius.

Escape needs **no rocket left over**: reach v_{esc} and you coast to infinity, arriving with zero speed.

The energy of two point masses

Potential energy

The **gravitational potential energy** of two point masses is $E_p = -\frac{GMm}{r}$ – negative, because the zero is placed at infinity.

The constant

$G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$ is the **universal gravitational constant** – the same everywhere, which is what “universal” asserts.

Escape speed

Escape needs $\frac{1}{2}mv^2 \geq \frac{GMm}{r}$, so $v_{\text{esc}} = \sqrt{2GM/r}$ – independent of the escaping mass.

The minus sign is not decoration: it says work must be *done* to separate the masses, and it is what makes a bound orbit have negative total energy.

Exercises 2.1-2.3

2.1 State Newton's law of gravitation.

[2]

2.2 State the value and unit of the gravitational constant G .

[1]

2.3 Two 1000 kg masses are 2.0 m apart. Find the gravitational force between them. [2]

Exercises 2.1·2.3

2.1 State Newton's law of gravitation.

[2]

2.1

- the gravitational force between two **point masses** is **proportional to the product of the masses**
 - and **inversely proportional to the square of their separation**
-

2.2 State the value and unit of the gravitational constant G .

[1]

2.2

- $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$
-

2.3 Two 1000 kg masses are 2.0 m apart. Find the gravitational force between them. [2]

2.3

- $F = Gm_1m_2/r^2$

$$F = \frac{Gm_1m_2}{r^2} = \frac{(6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2})(1000 \text{ kg})(1000 \text{ kg})}{(2.0 \text{ m})^2} = \mathbf{1.67 \times 10^{-5} \text{ N}}$$

Exercises 2.4·3.1

2.4 State **two** features of a **geostationary** orbit.

[2]

3.1 If the distance between two masses is **doubled**, the gravitational force becomes: [1]

☐

A twice as large

B half as large

C one quarter as large

D four times as large

Exercises 2.4·3.1

2.4 State **two** features of a **geostationary** orbit.

[2]

2.4

- any two: period of **24 h**; orbits **west to east**; stays **above the equator**; appears fixed over one point on Earth

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☐

A twice as large

B half as large

C one quarter as large

D four times as large

3.1 C

- $F \propto 1/r^2$, so doubling r makes F **one quarter**
- **A** would be $1/r$; **B** is twice as far so half the force; **D** is four times larger

Exercises 3.2·3.3

3.2 Calculate the gravitational field strength at the Earth's surface. ($M = 6.0 \times 10^{24}$ kg, $R = 6.4 \times 10^6$ m, $G = 6.67 \times 10^{-11}$.) [3]

3.3 A satellite moves in a circular orbit of radius r around a planet of mass M . Show that gravity provides the centripetal force, and derive an expression linking the orbital **period** T to r . [4]

Exercises 3.2·3.3

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3.2

- $g = GM/R^2$

$$g = \frac{GM}{R^2} = \frac{(6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2})(6.0 \times 10^{24} \text{ kg})}{(6.4 \times 10^6 \text{ m})^2} = \mathbf{9.8 \text{ N kg}^{-1}}$$

3.3 A satellite moves in a circular orbit of radius r around a planet of mass M . Show that gravity provides the centripetal force, and derive an expression linking the orbital period T to r . [4]

3.3

- gravity provides the centripetal force: $GMm/r^2 = mv^2/r$
- $v = 2\pi r/T$

$$\frac{GM}{r^2} = \frac{4\pi^2 r}{T^2}$$

- $T^2 = 4\pi^2 r^3/(GM)$

Exercise 3.4

3.4 State why a **geostationary** satellite must orbit directly above the **equator**. [1]

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3.4

- its orbital plane must contain the **Earth's centre**; only an equatorial orbit keeps it over a **fixed point** as the Earth spins