

Exercise walk-through

12.1

Kinematics of uniform circular motion

A-Level Physics

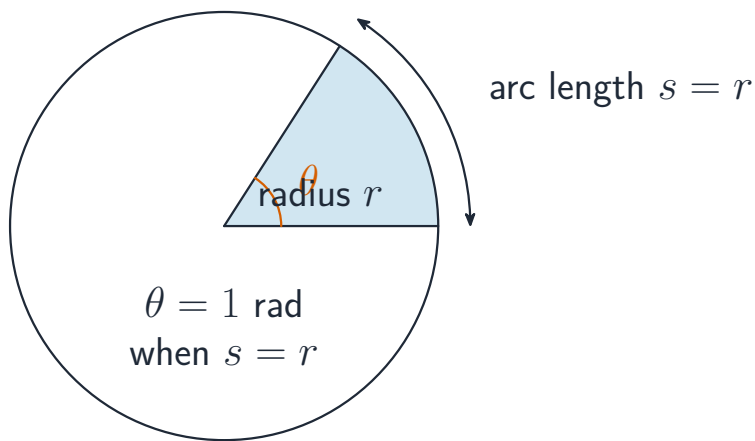
Angles in radians

One **radian** is the angle whose **arc** equals the radius:

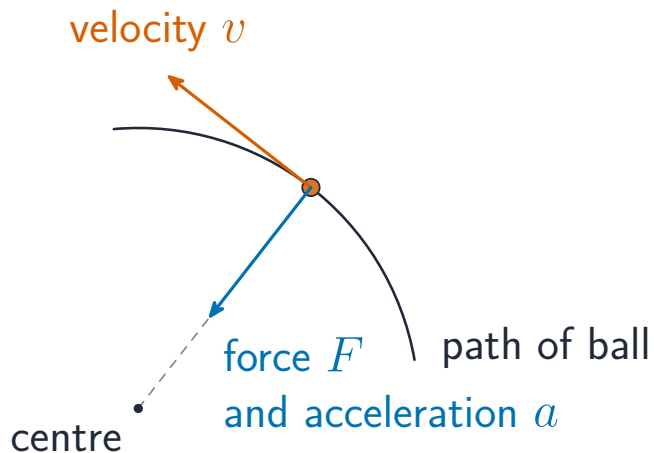
$$\theta = \frac{s}{r}$$

- a ratio of lengths $\Rightarrow$ **no unit**
- full circle = 2π rad; half = π
- $1 \text{ rad} = 180^\circ / \pi \approx 57.3^\circ$

Put the calculator in **radian** mode for this whole topic – degree mode silently gives wrong answers.



Centripetal acceleration



Constant **speed**, changing **velocity** – the direction keeps turning, so there *is* an acceleration:

$$a = \frac{v^2}{r} = r\omega^2$$

(equal, because $v = r\omega$ – pick the form matching your data).

a is **perpendicular** to v at every instant. Any part **along** the motion would change the speed.

Centripetal force



Circular motion at constant speed is still a **changing velocity** – the direction turns – so there is an acceleration, and Newton's second law radially gives:

$$F = ma = \frac{mv^2}{r} = mr\omega^2$$

The **centripetal force** is **not a new force**. It is the **net result** of the real forces – always say **which** one provides it.

Exercises 2.1-2.2

2.1 Define the **radian**.

[1]

2.2 Convert 90° to **radians**.

[1]

Exercises 2.1·2.2

2.1 Define the **radian**.

[1]

2.1

- the angle subtended at the centre of a circle by an arc whose **length equals the radius**

2.2 Convert 90° to **radians**.

[1]

2.2

- a full circle is $2\pi \text{ rad} = 360^\circ$

$$\theta = 90^\circ \times \frac{2\pi \text{ rad}}{360^\circ} = \frac{\pi}{2} \text{ rad} \approx 1.57 \text{ rad}$$

Exercises 2.3-2.4

2.3 A wheel turns at 2.0 revolutions per second. Find its **angular speed**. [2]

2.4 Two pulleys are linked by a belt. State what is **equal** for the two pulleys. [1]

Exercises 2.3·2.4

2.3 A wheel turns at 2.0 revolutions per second. Find its **angular speed**. [2]

2.3

- $f = 2.0 \text{ rev s}^{-1}; \omega = 2\pi f$

$$\omega = 2\pi f = 2\pi \times 2.0 \text{ s}^{-1} = 4\pi \text{ rad s}^{-1} = \mathbf{12.6 \text{ rad s}^{-1}}$$

2.4 Two pulleys are linked by a belt. State what is **equal** for the two pulleys. [1]

2.4

- their **rim speeds** ($v = r\omega$) are equal (the belt does not slip)

Exercises 3.1·3.2

3.1 A full circle, in radians, is:

[1]

☐

A π

B 2π

C 360

D 1

3.2 A car travels around a circular track of radius 50 m at $20 \frac{\text{m}}{\text{s}}$. Find its **angular speed**.

[2]

Exercises 3.1·3.2

3.1 A full circle, in radians, is:

[1]

☐

A π

B 2π

C 360

D 1

3.1 B

- a full circle is 2π radians
- **A** is a half-turn; **C** is degrees; **D** is one radian, not a full turn

3.2 A car travels around a circular track of radius 50 m at $20 \frac{\text{m}}{\text{s}}$. Find its **angular speed**. [2]

3.2

- $v = 20 \frac{\text{m}}{\text{s}}$, $r = 50 \text{ m}$; $\omega = v/r$

$$\omega = \frac{v}{r} = \frac{20 \frac{\text{m}}{\text{s}}}{50 \text{ m}} = \mathbf{0.40 \text{ rad s}^{-1}}$$

Exercise 3.3 (a)

3.3 In a workshop, a motor turns a pulley of radius 25 mm at 1500 revolutions per minute. A tight belt runs from that pulley to a larger pulley of radius 75 mm on the shaft of a fan.

(a) Determine the angular speed of the **motor pulley**, in rad s^{-1} . [2]

Exercise 3.3 (a)

3.3 In a workshop, a motor turns a pulley of radius 25 mm at 1500 revolutions per minute. A tight belt runs from that pulley to a larger pulley of radius 75 mm on the shaft of a fan.

(a) Determine the angular speed of the **motor pulley**, in rad s^{-1} . [2]

(a) 1500 revolutions per minute = 25 revolutions per second, and one revolution is 2π rad

$$\omega = 2\pi f = 2\pi \times 25 \text{ s}^{-1} = \mathbf{157} \text{ rad s}^{-1}$$

Exercise 3.3 (b)

3.3 In a workshop, a motor turns a pulley of radius 25 mm at 1500 revolutions per minute. A tight belt runs from that pulley to a larger pulley of radius 75 mm on the shaft of a fan.

(b) The belt does not slip. Determine the angular speed of the **fan**, and state what stays the same for both pulleys. [3]

Exercise 3.3 (b)

3.3 In a workshop, a motor turns a pulley of radius 25 mm at 1500 revolutions per minute. A tight belt runs from that pulley to a larger pulley of radius 75 mm on the shaft of a fan.

(b) The belt does not slip. Determine the angular speed of the **fan**, and state what stays the same for both pulleys. [3]

(b) the belt gives both pulleys the same **rim speed** $v = r\omega$

- motor radius 25 mm = 0.025 m

$$v = r\omega = (0.025 \text{ m})(157 \text{ rad s}^{-1}) = 3.93 \frac{\text{m}}{\text{s}}$$

$$\omega_{\text{fan}} = \frac{v}{r} = \frac{3.93 \frac{\text{m}}{\text{s}}}{0.075 \text{ m}} = \mathbf{52 \text{ rad s}^{-1}}$$

- a third of the motor's, because its radius is three times larger

Exercise 3.4

3.4 Convert 1.5 rad to **degrees**.

[1]

Exercise 3.4

3.4 Convert 1.5 rad to **degrees**.

[1]

3.4

$$\theta = 1.5 \text{ rad} \times \frac{180^\circ}{\pi} = \mathbf{86^\circ}$$