

Exercise walk-through

10.2

Kirchhoff's laws

A-Level Physics

10.2 Kirchhoff's laws and combining resistors

Name: _____ Class: _____ Date: _____ **Total: 18 marks**

KEY WORDS Kirchhoff's first law
• series • parallel • junction • Kirchhoff's second law

Objective

Answer exam questions on **Kirchhoff's laws** : the two laws and the conservation law behind each, the derivations of the series and parallel resistance formulas, and simple series and parallel circuits. Syllabus 10.2, learning outcomes 1 to 6.

You must be able to:

- state Kirchhoff's first and second laws, and the conservation law each follows from
- derive $R = R_1 + R_2$ for resistors in series and $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ for resistors in parallel, naming the law at each step
- use both formulas, and find the current at a junction

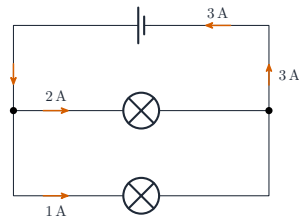
- find the current and the p.d.s in a series circuit

1 Key knowledge and worked examples

■ Key knowledge: use these earlier methods

- **NTC thermistor:** resistance falls as temperature rises. In a potential divider, $V_{\text{out}} = V_{\text{in}} R_{\text{out}} / (R_1 + R_2)$; identify which resistor the output is across before predicting a change.
- **Source with internal resistance:** $V = \mathcal{E} - Ir$; $\mathcal{E} = I(R + r)$ for a single external resistance R . A larger current causes a larger lost voltage Ir .
- Series resistances add; parallel branches share voltage and their currents add.

■ Key knowledge: Kirchhoff's laws



at each junction: $3\text{ A} = 2\text{ A} + 1\text{ A}$

- **First law.** At any **junction**, the total current in equals the total current out. Charge cannot pile up at a point, so this is conservation of charge.
- **Second law.** Round any closed loop, the total **electromotive force** equals the total p.d. across the components. A unit of charge gives up exactly the energy it gained, so this is conservation of energy.
- The pairing is examined almost every session, and the two are easy to swap. First law $\rightarrow$ charge. Second law $\rightarrow$ energy.

Combination	Same for each resistor	Combined resistance
series	current	$R = R_1 + R_2 + \dots$: always larger than the largest
parallel	p.d.	$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \dots$: always smaller than the smallest

- two equal resistors R in parallel give $\frac{R}{2}$; n equal ones give $\frac{R}{n}$

■ Worked example: deriving the series formula



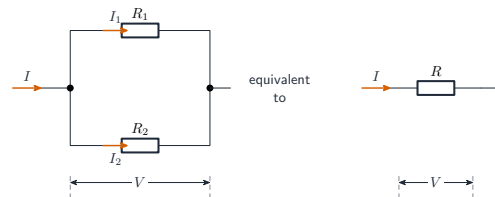
A derivation must name the law at each step; that is what earns the marks.

1. There is no junction between the resistors, so by the **first law** the same current

I is in both.

- By the **second law**, the p.d. across the combination is the sum of the p.d.s:
 $V = V_1 + V_2$.
- Each p.d. is IR , so $IR = IR_1 + IR_2$.
- Divide every term by I : $R = R_1 + R_2$.

■ **Worked example: deriving the parallel formula**



- Each resistor forms its own loop with the source, so by the **second law** the same p.d. V is across both.
- By the **first law** the current arriving at the junction is shared: $I = I_1 + I_2$.
- Each current is V/R , so $\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2}$.
- Divide every term by V : $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$.

■ **Worked example: a series circuit**

A 12 V battery of negligible internal resistance drives a current through a 3.0 Ω and a 6.0 Ω resistor in series.

- Combined resistance** – in series the resistances add:

$$R = R_1 + R_2 = 3.0 \, \Omega + 6.0 \, \Omega = 9.0 \, \Omega$$

- Current** – from $R = \frac{V}{I}$; it is the same in both resistors (first law):

$$I = \frac{V}{R} = \frac{12 \, \text{V}}{9.0 \, \Omega} = 1.33 \, \text{A}$$

- P.d.s** – $V = IR$ for each resistor; the two add to the supply's 12 V (second law):

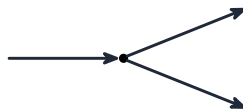
$$V_1 = IR_1 = (1.33 \, \text{A})(3.0 \, \Omega) = 4.0 \, \text{V}$$

$$V_2 = IR_2 = (1.33 \, \text{A})(6.0 \, \Omega) = 8.0 \, \text{V}$$

Exercises 2.1-2.2

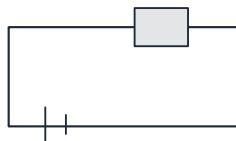
2.1 State **Kirchhoff's first law** and which conservation law it follows from.

[2]



2.2 State **Kirchhoff's second law** and which conservation law it follows from.

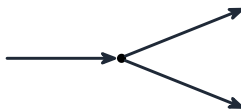
[2]



Exercises 2.1·2.2

2.1 State **Kirchhoff's first law** and which conservation law it follows from.

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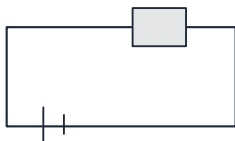


2.1

- the **total current into a junction equals the total current out**
- it follows from conservation of **charge**

2.2 State **Kirchhoff's second law** and which conservation law it follows from.

[2]



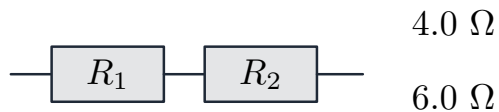
2.2

- around any closed loop, the **total e.m.f. equals the total p.d.**
- it follows from conservation of **energy**

Exercise 2.3

2.3 Find the combined resistance of $4.0\ \Omega$ and $6.0\ \Omega$ in **series**.

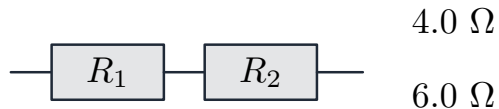
[1]



Exercise 2.3

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2.3

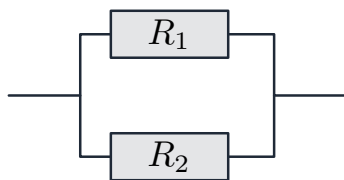
- series: $R = R_1 + R_2$

$$R = R_1 + R_2 = 4.0\ \Omega + 6.0\ \Omega = \mathbf{10\ \Omega}$$

Exercise 2.4

2.4 Find the combined resistance of two $4.0\ \Omega$ resistors in **parallel**.

[2]

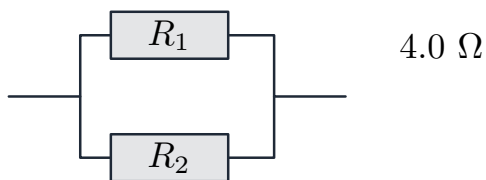


$4.0\ \Omega$

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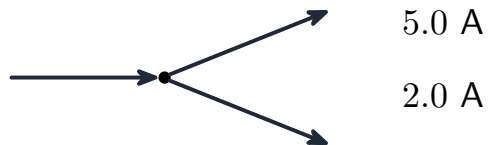
2.4

- parallel: $1/R = 1/R_1 + 1/R_2$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{4.0 \, \Omega} + \frac{1}{4.0 \, \Omega} = \frac{1}{2.0 \, \Omega} \Rightarrow R = \mathbf{2.0 \, \Omega}$$

Exercise 2.5

2.5 At a junction, 5.0 A enters and one branch carries 2.0 A away. Find the current in the other branch. [1]



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2.5

- Kirchhoff 1: current in = current out

$$I = I_{\text{in}} - I_1 = 5.0 \text{ A} - 2.0 \text{ A} = \mathbf{3.0 \text{ A}}$$

Exercise 3.1

3.1 Three $6.0\ \Omega$ resistors in **parallel** have a combined resistance of:

[1]

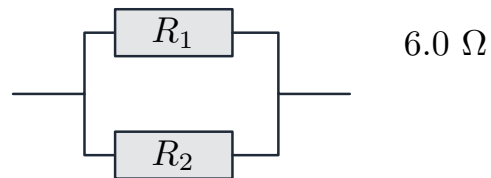
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A $2.0\ \Omega$

B $6.0\ \Omega$

C $18\ \Omega$

D $0.50\ \Omega$



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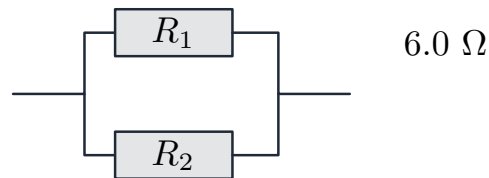
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A $2.0\ \Omega$

B $6.0\ \Omega$

C $18\ \Omega$

D $0.50\ \Omega$



3.1 A

- three equal resistors in parallel: $R = R_1/3$

$$R = \frac{R_1}{3} = \frac{6.0\ \Omega}{3} = \mathbf{2.0\ \Omega}$$

- **B** is one resistor; **C** is series; **D** would be twelve in parallel

Exercise 3.2

3.2 Two resistors of resistances $2R$ and $3R$ are connected in parallel. What is their combined resistance? [1]

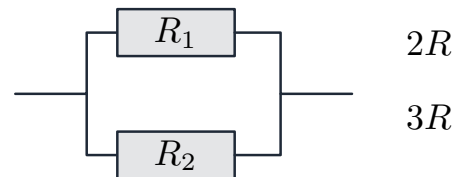
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A $\frac{5}{6}R$

B $\frac{6}{5}R$

C $\frac{5}{2}R$

D $5R$



Exercise 3.2

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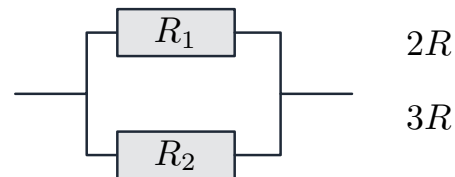
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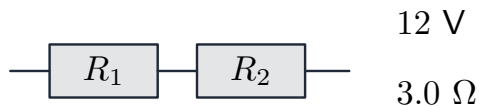


3.2 B

- parallel formula: $\frac{1}{R_{\text{total}}} = \frac{1}{2R} + \frac{1}{3R} = \frac{5}{6R}$, so $R_{\text{total}} = \frac{6}{5}R$
- D** adds them as if in series; **A** forgets to take the reciprocal

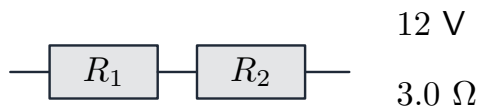
Exercise 3.3

3.3 A 12 V battery drives current through a $3.0\ \Omega$ and a $6.0\ \Omega$ resistor in **series**. Calculate the current and the p.d. across each resistor. [4]



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3.3

- in series the resistances add

Full solution 3.3

3.3

- in series the resistances add

$$R = R_3 + R_6 = 3.0 \, \Omega + 6.0 \, \Omega = 9.0 \, \Omega$$

- the current from $R = \frac{V}{I}$

$$I = \frac{V}{R} = \frac{12 \, \text{V}}{9.0 \, \Omega} = \mathbf{1.33 \, \text{A}}$$

- the p.d. across each resistor from $V = IR$

$$V_3 = IR_3 = (1.33 \, \text{A})(3.0 \, \Omega) = \mathbf{4.0 \, \text{V}}$$

$$V_6 = IR_6 = (1.33 \, \text{A})(6.0 \, \Omega) = \mathbf{8.0 \, \text{V}}$$

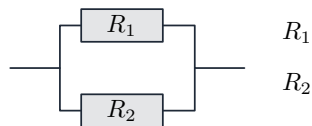
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3.4 Two resistors of resistances R_1 and R_2 are connected in parallel. Use Kirchhoff's laws to show that the combined resistance R is given by

$$R = \frac{R_1 R_2}{R_1 + R_2}.$$

Name the law used at each step.

[4]



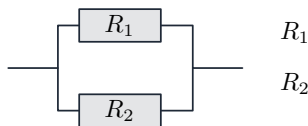
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Name the law used at each step.

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3.4

- by **Kirchhoff's second law** each resistor is in its own loop with the source, so the same p.d. V is across both

Full solution on next slide

Full solution 3.4

3.4

- by **Kirchhoff's second law** each resistor is in its own loop with the source, so the same p.d. V is across both
- by **Kirchhoff's first law** the currents at the junction add: $I = I_1 + I_2$
- substitute $I = V/R$ for each: $\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2}$, so $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$
- put the right-hand side over a common denominator and invert:

$$\frac{1}{R} = \frac{R_2 + R_1}{R_1 R_2} \Rightarrow R = \frac{R_1 R_2}{R_1 + R_2}$$