

Exercise walk-through

10.2.1

Solving circuits with Kirchhoff's laws

A-Level Physics

Sheet 10.2.1 – what you need to know

You must be able to:

- reduce a network one step at a time
- share the current at a junction and the p.d. round a loop
- decide which resistor dissipates the most **power**
- handle two cells in one loop

What the exam asks

“Show that the combined resistance R of the network is given by $R = R_1 + R_2 + R_3$.” [2]

“State and explain which resistor dissipates more power.” [3]

“State and explain the effect, if any, of this change on the current in the cell.” [2]

Practise on exercise sheet 10.2.1, then try the real exam questions on past-paper sheet 10.2.1.

Solving a circuit

1

label every current
& direction

2

junction rule at each node

3

loop rule around each loop

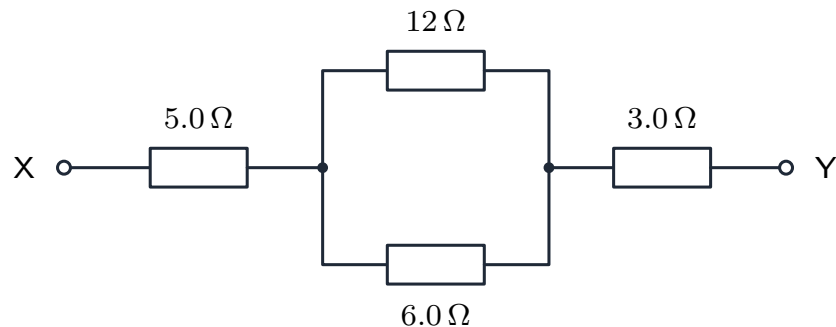
4

$V = IR$ for each resistor

5. solve the equations together

In a **symmetric** network, spot branches carrying equal currents – and remember the branch with the most current dissipates the most **power**, since $P = I^2 R$.

Reduce a network one step at a time



1. The parallel pair

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{12\ \Omega} + \frac{1}{6.0\ \Omega} = \frac{3}{12\ \Omega},$$

so $R_p = 4.0\ \Omega$

2. Then the three in series

$$\begin{aligned} R_{XY} &= R_a + R_p + R_b \\ &= 5.0\ \Omega + 4.0\ \Omega + 3.0\ \Omega = 12\ \Omega \end{aligned}$$

Check: Parallel: less than the smallest resistor. Series: more than the largest.

Which resistor dissipates the most power?

In series: the same current

$$I = \frac{V}{R_4 + R_8} = \frac{12 \text{ V}}{4.0 \, \Omega + 8.0 \, \Omega} = 1.0 \text{ A}$$

$$P_4 = I^2 R_4 = (1.0 \text{ A})^2 (4.0 \, \Omega) = 4.0 \text{ W}$$

$$P_8 = I^2 R_8 = (1.0 \text{ A})^2 (8.0 \, \Omega) = 8.0 \text{ W}$$

Same I and $P = I^2 R$: the **larger** resistor wins.

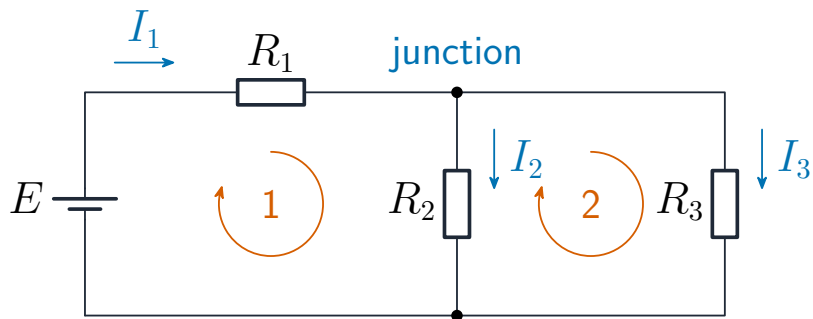
In parallel: the same p.d.

$$P_4 = \frac{V^2}{R_4} = \frac{(12 \text{ V})^2}{4.0 \, \Omega} = 36 \text{ W}$$

$$P_8 = \frac{V^2}{R_8} = \frac{(12 \text{ V})^2}{8.0 \, \Omega} = 18 \text{ W}$$

Same V and $P = \frac{V^2}{R}$: the **smaller** resistor wins.

Two loops: share the current, then the p.d.



$$E = 9.0 \text{ V}, R_1 = 2.0 \Omega, R_2 = R_3 = 6.0 \Omega$$

Find I_1 , I_2 and I_3

first law at the junction: $I_1 = I_2 + I_3$

$$\frac{1}{R_p} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{2}{6.0 \Omega}, \text{ so } R_p = 3.0 \Omega$$

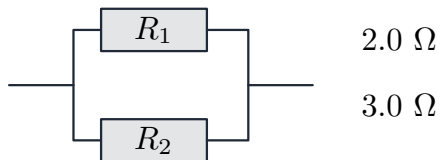
$$I_1 = \frac{E}{R_1 + R_p} = \frac{9.0 \text{ V}}{2.0 \Omega + 3.0 \Omega} = 1.8 \text{ A}$$

$$V_p = I_1 R_p = (1.8 \text{ A})(3.0 \Omega) = 5.4 \text{ V}$$

$$I_2 = \frac{V_p}{R_2} = \frac{5.4 \text{ V}}{6.0 \Omega} = 0.90 \text{ A} = I_3$$

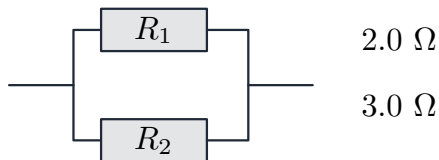
Exercise 2.1

2.1 Calculate the combined resistance of $2.0\ \Omega$, $3.0\ \Omega$ and $6.0\ \Omega$ connected in parallel.[2]



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2.1

- in parallel the reciprocals add

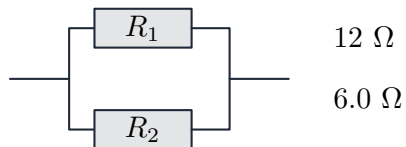
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R} = \frac{1}{2.0\ \Omega} + \frac{1}{3.0\ \Omega} + \frac{1}{6.0\ \Omega} = 1.0\ \Omega^{-1} \Rightarrow R = \mathbf{1.0\ \Omega}$$

- the answer is smaller than the smallest resistor, as it must be

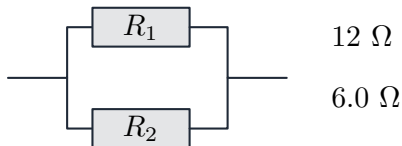
Exercise 2.2

2.2 For the network between X and Y in the worked example above, the $12\ \Omega$ resistor is replaced by a second $6.0\ \Omega$ resistor. Calculate the new resistance between X and Y. [3]



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2.2

- the parallel pair is now two equal resistors, so it has half the resistance of one

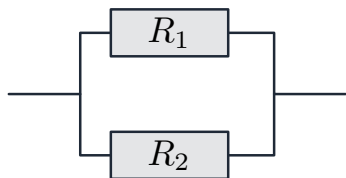
$$R_p = \frac{R}{2} = \frac{6.0\ \Omega}{2} = 3.0\ \Omega$$

- in series with the others the resistances add

$$R_{XY} = R_a + R_p + R_b = 5.0\ \Omega + 3.0\ \Omega + 3.0\ \Omega = \mathbf{11\ \Omega}$$

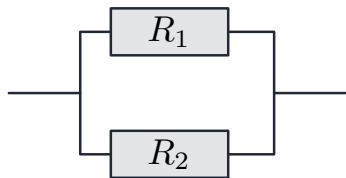
Exercise 2.3

2.3 Explain why connecting a second resistor in parallel with the first always **decreases** the resistance between the same two points. [2]



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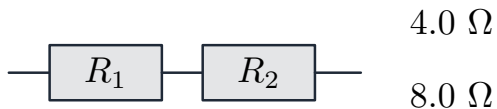


2.3

- the second resistor gives the charge an extra path, so for the same p.d. the total current is larger
- $R = V/I$, so a larger current for the same p.d. means a smaller resistance

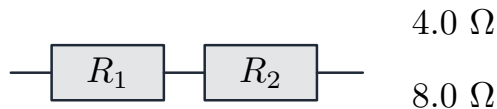
Exercise 2.4

2.4 A $4.0\ \Omega$ and an $8.0\ \Omega$ resistor are connected in **series** across a supply. State which dissipates the greater power, and by what factor. [2]



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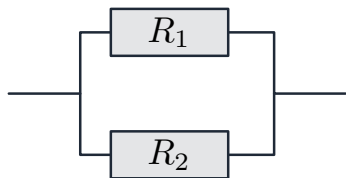


2.4

- in series the current is the same in both, and $P = I^2 R$
- the **$8.0\ \Omega$** resistor, dissipating **twice** the power of the $4.0\ \Omega$

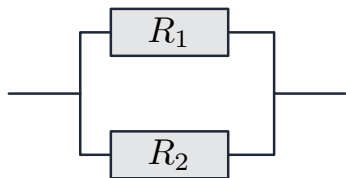
Exercise 2.5

2.5 The same two resistors are now connected in **parallel** across the same supply. State which dissipates the greater power, and explain why the answer has changed. [2]



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2.5 The same two resistors are now connected in **parallel** across the same supply. State which dissipates the greater power, and explain why the answer has changed. [2]



2.5

- in parallel the p.d. is the same across both, and $P = V^2/R$, so the **4.0 Ω** resistor dissipates the greater power
- the answer changes because the resistors now share the p.d. instead of the current

Exercise 3.1

3.1 A $6.0\ \Omega$ resistor and a $3.0\ \Omega$ resistor are connected in parallel. This combination is in series with a $4.0\ \Omega$ resistor across a supply of e.m.f. $12\ \text{V}$ and negligible internal resistance. What is the current from the supply? [1]

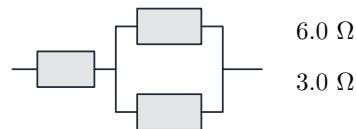
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A $0.50\ \text{A}$

B $1.0\ \text{A}$

C $2.0\ \text{A}$

D $3.0\ \text{A}$

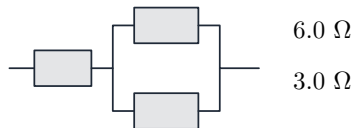


Exercise 3.1

3.1 A $6.0\ \Omega$ resistor and a $3.0\ \Omega$ resistor are connected in parallel. This combination is in series with a $4.0\ \Omega$ resistor across a supply of e.m.f. $12\ \text{V}$ and negligible internal resistance. What is the current from the supply? [1]

☐

- A** $0.50\ \text{A}$
- B** $1.0\ \text{A}$
- C** $2.0\ \text{A}$
- D** $3.0\ \text{A}$



3.1 C

- the parallel pair: the reciprocals add

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{6.0\ \Omega} + \frac{1}{3.0\ \Omega} = \frac{1}{2.0\ \Omega} \Rightarrow R_p = 2.0\ \Omega$$

- in series with the $4.0\ \Omega$ resistor, then the current from $R = \frac{V}{I}$

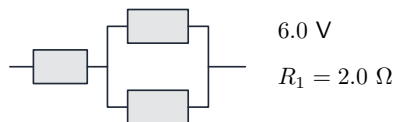
$$R = R_s + R_p = 4.0\ \Omega + 2.0\ \Omega = 6.0\ \Omega$$

$$I = \frac{E}{R} = \frac{12\ \text{V}}{6.0\ \Omega} = 2.0\ \text{A}$$

Exercise 3.2 (a)

3.2 A battery of e.m.f. 6.0 V and negligible internal resistance is connected to $R_1 = 2.0\ \Omega$ in series with two resistors $R_2 = 4.0\ \Omega$ and $R_3 = 4.0\ \Omega$ in **parallel**.

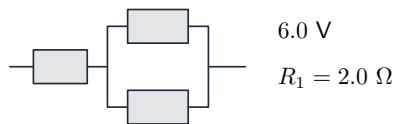
(a) Calculate the total resistance and the current from the battery. [3]



Exercise 3.2 (a)

3.2 A battery of e.m.f. 6.0 V and negligible internal resistance is connected to $R_1 = 2.0 \, \Omega$ in series with two resistors $R_2 = 4.0 \, \Omega$ and $R_3 = 4.0 \, \Omega$ in **parallel**.

(a) Calculate the total resistance and the current from the battery. [3]



(a) the parallel pair: the reciprocals add

$$\frac{1}{R_p} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{4.0 \, \Omega} + \frac{1}{4.0 \, \Omega} = \frac{1}{2.0 \, \Omega} \Rightarrow R_p = 2.0 \, \Omega$$

- in series with R_1 the resistances add

$$R = R_1 + R_p = 2.0 \, \Omega + 2.0 \, \Omega = \mathbf{4.0 \, \Omega}$$

- the current from the battery

$$I = \frac{E}{R} = \frac{6.0 \, \text{V}}{4.0 \, \Omega} = \mathbf{1.5 \, \text{A}}$$

Exercise 3.2 (b)

3.2 A battery of e.m.f. 6.0 V and negligible internal resistance is connected to $R_1 = 2.0 \Omega$ in series with two resistors $R_2 = 4.0 \Omega$ and $R_3 = 4.0 \Omega$ in **parallel**.

(b) Calculate the current in **each** of R_2 and R_3 .

[3]

Exercise 3.2 (b)

3.2 A battery of e.m.f. 6.0 V and negligible internal resistance is connected to $R_1 = 2.0\ \Omega$ in series with two resistors $R_2 = 4.0\ \Omega$ and $R_3 = 4.0\ \Omega$ in **parallel**.

(b) Calculate the current in **each** of R_2 and R_3 . [3]

(b) the p.d. across the parallel pair, $V_p = IR_p$

$$V_p = IR_p = (1.5\text{ A})(2.0\ \Omega) = 3.0\text{ V}$$

- each branch has this p.d. across it

$$I_2 = \frac{V_p}{R_2} = \frac{3.0\text{ V}}{4.0\ \Omega} = \mathbf{0.75\text{ A}}$$

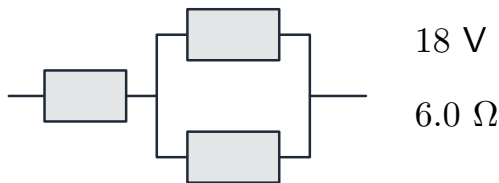
- the current in R_3 is the same, **0.75 A**, because $R_2 = R_3$

Exercise 3.3 (a)

3.3 A battery of e.m.f. 18 V and negligible internal resistance is connected to a $6.0\ \Omega$ resistor in series with a parallel combination of a $9.0\ \Omega$ resistor and an $18\ \Omega$ resistor.

(a) Show that the resistance of the parallel combination is $6.0\ \Omega$.

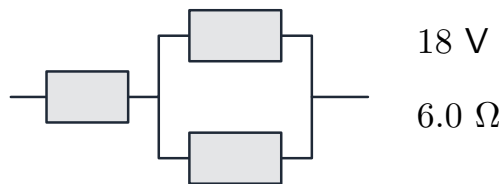
[2]



Exercise 3.3 (a)

3.3 A battery of e.m.f. 18 V and negligible internal resistance is connected to a $6.0\ \Omega$ resistor in series with a parallel combination of a $9.0\ \Omega$ resistor and an $18\ \Omega$ resistor.

(a) Show that the resistance of the parallel combination is $6.0\ \Omega$. [2]



(a) for resistors in parallel the reciprocals add

$$\frac{1}{R_p} = \frac{1}{R_9} + \frac{1}{R_{18}} = \frac{1}{9.0\ \Omega} + \frac{1}{18\ \Omega} = \frac{3}{18\ \Omega} \Rightarrow R_p = \mathbf{6.0\ \Omega}$$

Exercise 3.3 (b)

3.3 A battery of e.m.f. 18 V and negligible internal resistance is connected to a $6.0\ \Omega$ resistor in series with a parallel combination of a $9.0\ \Omega$ resistor and an $18\ \Omega$ resistor.

(b) Calculate the current in the battery.

[2]

Exercise 3.3 (b)

3.3 A battery of e.m.f. 18 V and negligible internal resistance is connected to a $6.0\ \Omega$ resistor in series with a parallel combination of a $9.0\ \Omega$ resistor and an $18\ \Omega$ resistor.

(b) Calculate the current in the battery. [2]

(b) in series with the $6.0\ \Omega$ resistor the resistances add

$$R = R_6 + R_p = 6.0\ \Omega + 6.0\ \Omega = 12\ \Omega$$

$$I = \frac{E}{R} = \frac{18\ \text{V}}{12\ \Omega} = \mathbf{1.5\ \text{A}}$$

Exercise 3.3 (c)

3.3 A battery of e.m.f. 18 V and negligible internal resistance is connected to a $6.0\ \Omega$ resistor in series with a parallel combination of a $9.0\ \Omega$ resistor and an $18\ \Omega$ resistor.

(c) Calculate the p.d. across the parallel combination and the current in each of its branches. [3]

Exercise 3.3 (c)

3.3 A battery of e.m.f. 18 V and negligible internal resistance is connected to a $6.0\ \Omega$ resistor in series with a parallel combination of a $9.0\ \Omega$ resistor and an $18\ \Omega$ resistor.

(c) Calculate the p.d. across the parallel combination and the current in each of its branches. [3]

(c) the p.d. across the combination, $V_p = IR_p$

$$V_p = IR_p = (1.5\ \text{A})(6.0\ \Omega) = \mathbf{9.0\ V}$$

- each branch has this p.d. across it, so $I = \frac{V}{R}$

$$I_9 = \frac{V_p}{R_9} = \frac{9.0\ \text{V}}{9.0\ \Omega} = \mathbf{1.0\ A}$$

$$I_{18} = \frac{V_p}{R_{18}} = \frac{9.0\ \text{V}}{18\ \Omega} = \mathbf{0.50\ A}$$

- check with the first law: $I = I_9 + I_{18} = 1.0\ \text{A} + 0.50\ \text{A} = 1.5\ \text{A}$, the current in the battery

Exercise 3.3 (d)

3.3 A battery of e.m.f. 18 V and negligible internal resistance is connected to a $6.0\ \Omega$ resistor in series with a parallel combination of a $9.0\ \Omega$ resistor and an $18\ \Omega$ resistor.

(d) State, with a reason, which of the three resistors dissipates the greatest power. [2]

Exercise 3.3 (d)

3.3 A battery of e.m.f. 18 V and negligible internal resistance is connected to a $6.0\ \Omega$ resistor in series with a parallel combination of a $9.0\ \Omega$ resistor and an $18\ \Omega$ resistor.

(d) State, with a reason, which of the three resistors dissipates the greatest power. [2]

(d) the **6.0** Ω resistor

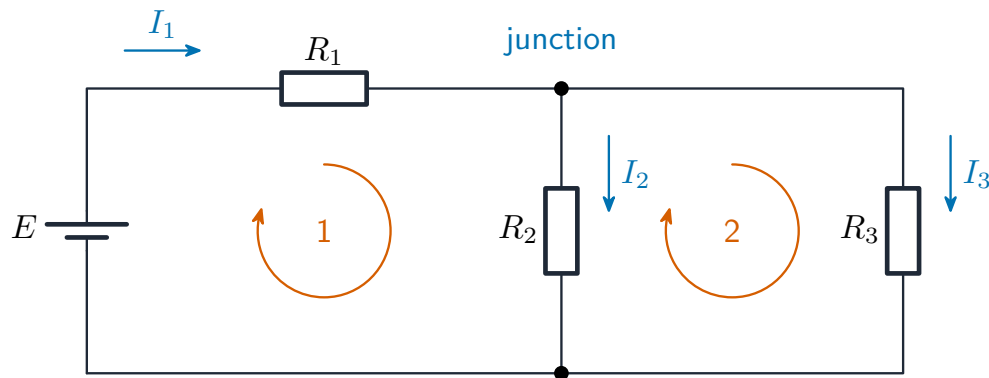
- it carries the whole 1.5 A while each branch carries only part of it

$$P_6 = I^2 R_6 = (1.5\ \text{A})^2 (6.0\ \Omega) = 13.5\ \text{W}$$

- against 9.0 W in the $9.0\ \Omega$ and 4.5 W in the $18\ \Omega$ resistor

Setup for exercise 3.4

3.4 A cell of e.m.f. 12 V and negligible internal resistance is connected to $R_1 = 3.0\ \Omega$ in series with a parallel combination of $R_2 = 12\ \Omega$ and $R_3 = 4.0\ \Omega$, as shown.



Exercise 3.4 (a)

- (a) Write down the equation that Kirchhoff's first law gives for the junction. [1]

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(a) Write down the equation that Kirchhoff's first law gives for the junction. [1]

(a) $I_1 = I_2 + I_3$

Exercise 3.4 (b)

(b) Calculate the current I_1 in the cell and the currents I_2 and I_3 in the two branches.[4]

Exercise 3.4 (b)

(b) Calculate the current I_1 in the cell and the currents I_2 and I_3 in the two branches.[4]

(b) the parallel pair: the reciprocals add

$$\frac{1}{R_p} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{12\ \Omega} + \frac{1}{4.0\ \Omega} = \frac{4}{12\ \Omega} \Rightarrow R_p = 3.0\ \Omega$$

- in series with R_1 the resistances add, then $I_1 = \frac{\varepsilon}{R}$

$$R = R_1 + R_p = 3.0\ \Omega + 3.0\ \Omega = 6.0\ \Omega$$

$$I_1 = \frac{\varepsilon}{R} = \frac{12\ \text{V}}{6.0\ \Omega} = \mathbf{2.0\ A}$$

- the p.d. across the pair, $V_p = I_1 R_p$

$$V_p = I_1 R_p = (2.0\ \text{A})(3.0\ \Omega) = 6.0\ \text{V}$$

- each branch has this p.d. across it

$$I_2 = \frac{V_p}{R_2} = \frac{6.0\ \text{V}}{12\ \Omega} = \mathbf{0.50\ A}$$

$$I_3 = \frac{V_p}{R_3} = \frac{6.0\ \text{V}}{4.0\ \Omega} = \mathbf{1.5\ A}$$

Exercise 3.4 (c)

(c) Calculate the p.d. across R_1 .

[2]

Exercise 3.4 (c)

(c) Calculate the p.d. across R_1 .

[2]

(c) $V_1 = I_1 R_1$

$$V_1 = I_1 R_1 = (2.0 \text{ A})(3.0 \text{ } \Omega) = \mathbf{6.0 \text{ V}}$$

Exercise 3.4 (d)

(d) Show that the total power dissipated in the three resistors equals the power supplied by the cell, and state which conservation law this illustrates. [3]

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(d) $P = I^2 R$ for each resistor

$$P_1 = I_1^2 R_1 = (2.0 \text{ A})^2 (3.0 \text{ } \Omega) = 12 \text{ W}$$

$$P_2 = I_2^2 R_2 = (0.50 \text{ A})^2 (12 \text{ } \Omega) = 3.0 \text{ W}$$

$$P_3 = I_3^2 R_3 = (1.5 \text{ A})^2 (4.0 \text{ } \Omega) = 9.0 \text{ W}$$

- the total dissipated

$$P = P_1 + P_2 + P_3 = 12 \text{ W} + 3.0 \text{ W} + 9.0 \text{ W} = \mathbf{24 \text{ W}}$$

- the cell supplies the same, illustrating **conservation of energy**, which is what Kirchhoff's second law expresses

$$P = \varepsilon I_1 = (12 \text{ V})(2.0 \text{ A}) = 24 \text{ W}$$