

Exercise walk-through

1.4

Scalars and vectors

A-Level Physics

1.4 Scalars and vectors

Name: _____ Class: _____ Date: _____ **Total: 34 marks**

KEY WORDS vector • scalar • component • perpendicular
• magnitude

Objective

Build the skills to answer the real exam questions on **scalars and vectors**. In the exam, 1.4 is tested by **multiple-choice** questions and by short parts inside longer questions: classify a quantity, find a **resultant**, or **resolve** a vector into components.

You must be able to:

- tell a **scalar** from a **vector** and complete a classification table
- find the **resultant** of two **perpendicular** vectors (size and direction)
- give the direction of $\vec{X} + \vec{Y}$ and $\vec{X} - \vec{Y}$
- find the resultant of **two equal forces** at an angle
- **resolve** any vector (a force *or* a velocity) into two perpendicular **components**
- add three or more **coplanar** vectors by components

- **sketch** each component of a projectile's velocity against time

1 Key knowledge and worked examples

Study these first. Each one shows the method for an exam question type used below – read them and you can do the Practice and Exam-style questions yourself.

■ A – Scalar or vector?

A scalar has size only; a vector has size **and** direction. **Quick test:** if “in which direction?” makes sense, it is a vector. In the exam you often tick a table:

Quantity	Scalar	Vector
speed	✓	
acceleration		✓
energy	✓	
momentum		✓

(Common scalars: mass, time, temperature, energy, distance, speed. Common vectors: displacement, velocity, acceleration, force, momentum.)

■ B – Add two perpendicular vectors

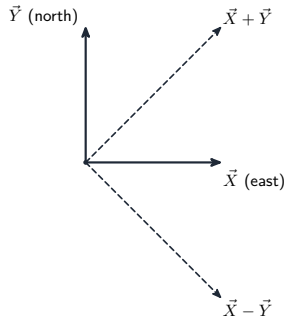
Draw them **tip to tail**; the resultant closes the triangle. For two vectors at 90° the size is $|\vec{R}| = \sqrt{X^2 + Y^2}$ and the direction is $\tan \theta = Y/X$.

Worked example. A swimmer heads north at $1.2 \frac{\text{m}}{\text{s}}$; the river flows east at $0.5 \frac{\text{m}}{\text{s}}$:

$$|\vec{R}| = \sqrt{1.2^2 + 0.5^2} = 1.3 \frac{\text{m}}{\text{s}}, \quad \tan \theta = \frac{0.5}{1.2} \Rightarrow \theta \approx 23^\circ \text{ east of north.}$$

■ C – Direction of $\vec{X} + \vec{Y}$ and $\vec{X} - \vec{Y}$

To **add**, place the arrows tip to tail. To **subtract**, reverse the second arrow first: $\vec{X} - \vec{Y} = \vec{X} + (-\vec{Y})$, where $-\vec{Y}$ is the same size as $\vec{Y}$ but points the opposite way.



With $\vec{X}$ east and $\vec{Y}$ north: $\vec{X} + \vec{Y}$ points **north-east**, and $\vec{X} - \vec{Y}$ points **south-east**.

■ C2 – Adding three or more coplanar vectors

Coplanar vectors all lie in the same flat plane – every vector in this course is coplanar. For three or more, drawing triangles gets messy, so **use components**: add all the x -parts, add all the y -parts, then recombine.

Worked example. Three coplanar forces act at a point: 2.0 N east, 3.0 N north, 4.0 N west.

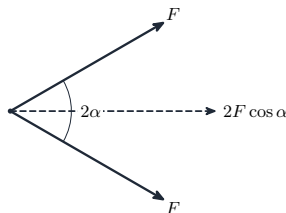
$$\sum x = 2.0 - 4.0 = -2.0 \text{ N}, \quad \sum y = +3.0 \text{ N}$$

$$F_{\text{res}} = \sqrt{2.0^2 + 3.0^2} = 3.6 \text{ N}, \quad \tan \theta = \frac{3.0}{2.0} \Rightarrow \theta = 56^\circ \text{ north of west}$$

Take one direction as positive and stay with it – a lost minus sign is the usual error.

■ **D – Two equal forces at an angle**

Two equal forces of size F with an angle 2α between them have a resultant of size $2F \cos \alpha$, along the line that **cuts the angle in half**.

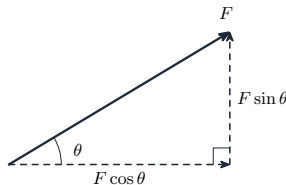


Worked example. Two 6.0 N forces act with 60° between them. Here $2\alpha = 60^\circ$, so $\alpha = 30^\circ$:

$$F_{\text{res}} = 2F \cos \alpha = 2(6.0) \cos 30^\circ = 10.4 \text{ N.}$$

■ **E – Resolve a vector into components**

Any vector $\vec{v}$ at angle θ to the horizontal splits into a horizontal part $v \cos \theta$ and a vertical part $v \sin \theta$ – this works for a force **or** a velocity.



Force example. A 50 N force at 30° : horizontal = $50 \cos 30^\circ = 43 \text{ N}$, vertical = $50 \sin 30^\circ = 25 \text{ N}$.

Velocity example. A ball kicked at $28 \frac{\text{m}}{\text{s}}$, 34° above the horizontal: $v_H = 28 \cos 34^\circ = 23 \frac{\text{m}}{\text{s}}$, $v_V = 28 \sin 34^\circ = 16 \frac{\text{m}}{\text{s}}$.

On a slope of angle θ , the weight splits into $F_W \sin \theta$ **along** the slope and $F_W \cos \theta$ **into** the slope.

■ F – The two components of a projectile, against time

Once the components are found, each one has its own simple graph – and the exam asks you to sketch both on one grid:

- **horizontal** v_H : a **horizontal straight line** (no horizontal force, so it never changes).
- **vertical** v_V : a **straight line sloping down**, starting at $+v \sin \theta$, crossing zero at the top of the flight, and reaching $-v \sin \theta$ on landing (taking up as positive).

The gradient of that second line is $-g$, and the time to the top is v_V/g . Label each line, and use the whole time axis you are given.

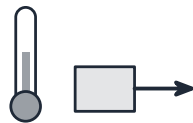
■ Key knowledge: a spherical volume with uncertainty

For a sphere, $V = 4\pi r^3/3$ gives $r = (3V/4\pi)^{1/3}$. The percentage uncertainty in r is one third of that in V ; retain extra digits until the final result. Example: $V = 36 \text{ cm}^3$ gives $r = (3 \times 36/4\pi)^{1/3} = 2.05 \text{ cm}$; a 3% volume uncertainty gives a 1% radius uncertainty.

Exercise 2.1

2.1 Complete the table by ticking **scalar** or **vector** for each quantity.

[3]

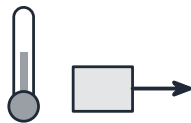


Quantity	Scalar	Vector
distance		
force		
temperature		
velocity		
power		
acceleration		

Exercise 2.1

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[3]



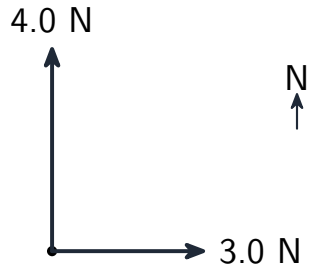
Quantity	Scalar	Vector
distance		
force		
temperature		
velocity		
power		
acceleration		

2.1

- scalar: size only; vector: size **and** direction
- distance **scalar**; force **vector**; temperature **scalar**; velocity **vector**; power **scalar**; acceleration **vector**

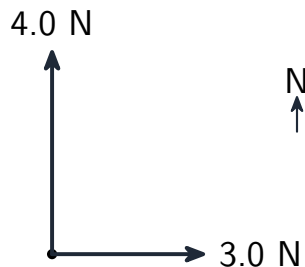
Exercise 2.2

2.2 Forces of 3.0 N east and 4.0 N north act at a point. Find the **magnitude** of the resultant. [2]



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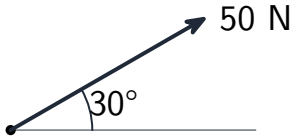
2.2

- the forces are perpendicular, so use Pythagoras

$$F_{\text{res}} = \sqrt{F_1^2 + F_2^2} = \sqrt{(3.0 \text{ N})^2 + (4.0 \text{ N})^2} = \sqrt{25 \text{ N}^2} = \mathbf{5.0 \text{ N}}$$

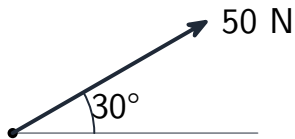
Exercise 2.3

2.3 Resolve a 50 N force acting at 30° to the horizontal into its horizontal and vertical components. [2]



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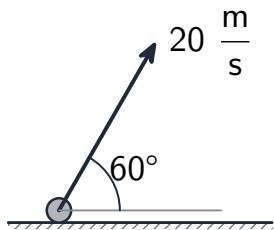


2.3

- horizontal is the adjacent side: $F \cos \theta$; vertical is the opposite side: $F \sin \theta$
- horizontal: $50 \text{ N} \times \cos 30^\circ = \mathbf{43 \text{ N}}$
- vertical: $50 \text{ N} \times \sin 30^\circ = \mathbf{25 \text{ N}}$

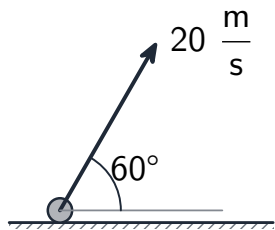
Exercise 2.4

2.4 A ball leaves the ground at $20 \frac{\text{m}}{\text{s}}$ at 60° above the horizontal. Find the **horizontal** and **vertical** components of its velocity. [2]



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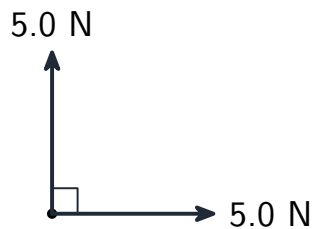


2.4

- $v_H = v \cos \theta, v_V = v \sin \theta$
- $v_H = 20 \frac{\text{m}}{\text{s}} \times \cos 60^\circ = \mathbf{10 \frac{m}{s}}$
- $v_V = 20 \frac{\text{m}}{\text{s}} \times \sin 60^\circ = \mathbf{17 \frac{m}{s}}$

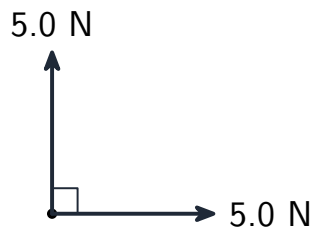
Exercise 2.5

2.5 Two forces, each 5.0 N, act at a point with 90° between them. Find the **magnitude** of the resultant. [2]



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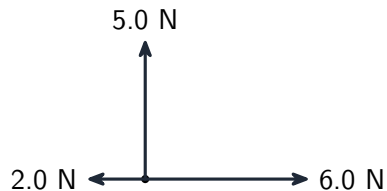
2.5

- equal forces at 90° : Pythagoras, or $F_{\text{res}} = 2F \cos 45^\circ$

$$F_{\text{res}} = \sqrt{F_1^2 + F_2^2} = \sqrt{(5.0 \text{ N})^2 + (5.0 \text{ N})^2} = \mathbf{7.1 \text{ N}}$$

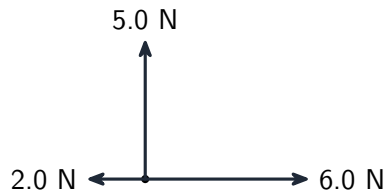
Exercise 2.6

2.6 Three **coplanar** forces act at a point: 6.0 N east, 2.0 N west and 5.0 N north. Find the **magnitude** and the **direction** of the resultant. [3]



Exercise 2.6

2.6 Three **coplanar** forces act at a point: 6.0 N east, 2.0 N west and 5.0 N north. Find the **magnitude** and the **direction** of the resultant. [3]



2.6

- take east and north as positive
- $\sum x = 6.0 \text{ N} - 2.0 \text{ N} = 4.0 \text{ N east}$; $\sum y = 5.0 \text{ N north}$

$$F_{\text{res}} = \sqrt{(\sum x)^2 + (\sum y)^2} = \sqrt{(4.0 \text{ N})^2 + (5.0 \text{ N})^2} = \mathbf{6.4 \text{ N}}$$

$$\tan \theta = \frac{\sum y}{\sum x} = \frac{5.0 \text{ N}}{4.0 \text{ N}} \Rightarrow \theta = \mathbf{51^\circ \text{ north of east}}$$

Exercise 3.1

3.1 Which of these is a **vector** quantity?

[1]

☐

A energy

B power

C displacement

D temperature

scalar

vector

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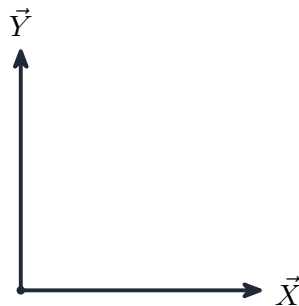
vector

3.1 C

- a vector needs a **direction** as well as a size
- **displacement** is “how far, and which way”
- **A, B, D** (energy, power, temperature) are scalars – *distance* is the scalar trap against displacement

Exercise 3.2

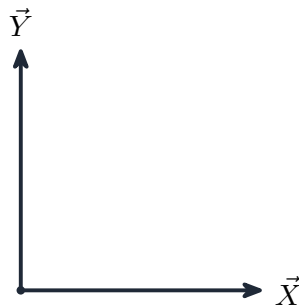
3.2 Two vectors $\vec{X}$ and $\vec{Y}$ are shown: $\vec{X}$ points east and $\vec{Y}$ points north. What are the directions of $\vec{X} + \vec{Y}$ and $\vec{X} - \vec{Y}$? [1]



- ☐ **A** $\vec{X} + \vec{Y}$ north-east; $\vec{X} - \vec{Y}$ south-east
B $\vec{X} + \vec{Y}$ north-east; $\vec{X} - \vec{Y}$ north-west
C $\vec{X} + \vec{Y}$ south-west; $\vec{X} - \vec{Y}$ south-east
D $\vec{X} + \vec{Y}$ south-east; $\vec{X} - \vec{Y}$ north-east

Exercise 3.2

3.2 Two vectors $\vec{X}$ and $\vec{Y}$ are shown: $\vec{X}$ points east and $\vec{Y}$ points north. What are the directions of $\vec{X} + \vec{Y}$ and $\vec{X} - \vec{Y}$? [1]



- ☐ A $\vec{X} + \vec{Y}$ north-east; $\vec{X} - \vec{Y}$ south-east
B $\vec{X} + \vec{Y}$ north-east; $\vec{X} - \vec{Y}$ north-west
C $\vec{X} + \vec{Y}$ south-west; $\vec{X} - \vec{Y}$ south-east
D $\vec{X} + \vec{Y}$ south-east; $\vec{X} - \vec{Y}$ north-east

3.2 A

- $\vec{X}$ east and $\vec{Y}$ north: tip-to-tail sum points **north-east**
- $\vec{X} - \vec{Y} = \vec{X} + (-\vec{Y})$: reverse $\vec{Y}$ to south, so the result points **south-east**

Full solution on next slide

Full solution 3.2

3.2 A

- $\vec{X}$ east and $\vec{Y}$ north: tip-to-tail sum points **north-east**
- $\vec{X} - \vec{Y} = \vec{X} + (-\vec{Y})$: reverse $\vec{Y}$ to south, so the result points **south-east**
- **B** keeps $\vec{Y}$ pointing north when subtracting

Exercise 3.3

3.3 Two forces of 6.0 N and 8.0 N act at a point at right angles to each other. What is the magnitude of the resultant force? [1]

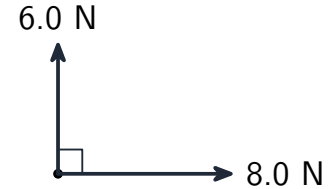
☐

A 2.0 N

B 10 N

C 14 N

D 48 N



Exercise 3.3

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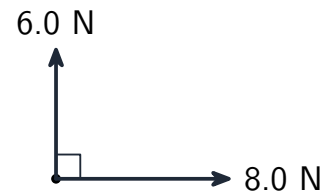
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A 2.0 N

B 10 N

C 14 N

D 48 N



3.3 B

- perpendicular forces: Pythagoras

$$F_{\text{res}} = \sqrt{F_1^2 + F_2^2} = \sqrt{(6.0 \text{ N})^2 + (8.0 \text{ N})^2} = \sqrt{100 \text{ N}^2} = \mathbf{10 \text{ N}}$$

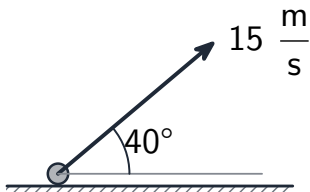
- A** is the difference; **C** is the sum; **D** is the product

Exercise 3.4 (a)

3.4 A stone is thrown with a velocity of $15 \frac{\text{m}}{\text{s}}$ at 40° above the horizontal.

(a) Find the **horizontal** and **vertical** components of this velocity.

[2]

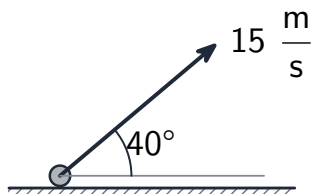


Exercise 3.4 (a)

3.4 A stone is thrown with a velocity of $15 \frac{\text{m}}{\text{s}}$ at 40° above the horizontal.

(a) Find the **horizontal** and **vertical** components of this velocity.

[2]



(a) resolve at 40° to the horizontal

- $v_H = 15 \frac{\text{m}}{\text{s}} \times \cos 40^\circ = \mathbf{11} \frac{\text{m}}{\text{s}}$
- $v_V = 15 \frac{\text{m}}{\text{s}} \times \sin 40^\circ = \mathbf{9.6} \frac{\text{m}}{\text{s}}$

Exercise 3.4 (b)

3.4 A stone is thrown with a velocity of $15 \frac{\text{m}}{\text{s}}$ at 40° above the horizontal.

(b) Air resistance is negligible. State which component stays **constant** during the flight, and why. [2]

Exercise 3.4 (b)

3.4 A stone is thrown with a velocity of $15 \frac{\text{m}}{\text{s}}$ at 40° above the horizontal.

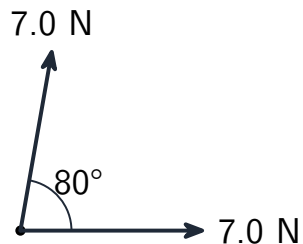
(b) Air resistance is negligible. State which component stays **constant** during the flight, and why. [2]

(b) the **horizontal** component stays constant

- no air resistance $\Rightarrow$ no horizontal force $\Rightarrow$ no horizontal acceleration

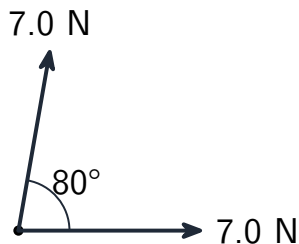
Exercise 3.5

3.5 Two forces, each 7.0 N, act at a point with 80° between them. Find the **magnitude** of the resultant. [2]



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3.5

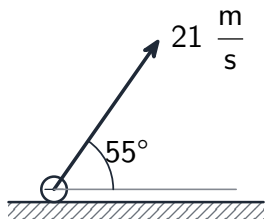
- two equal forces: $2\alpha = 80^\circ$ so $\alpha = 40^\circ$

$$F_{\text{res}} = 2F \cos \alpha = 2 \times 7.0 \text{ N} \times \cos 40^\circ = \mathbf{10.7 \text{ N}}$$

Exercise 3.6 (a)

3.6 A player kicks a ball from level ground with a velocity of $21 \frac{\text{m}}{\text{s}}$ at 55° to the horizontal. Air resistance is negligible.

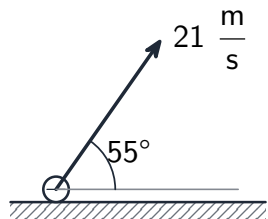
(a) **Calculate** the horizontal component v_H and the vertical component v_V of the velocity of the ball as it leaves the ground. [2]



Exercise 3.6 (a)

3.6 A player kicks a ball from level ground with a velocity of $21 \frac{\text{m}}{\text{s}}$ at 55° to the horizontal. Air resistance is negligible.

(a) **Calculate** the horizontal component v_H and the vertical component v_V of the velocity of the ball as it leaves the ground. [2]



(a)

- $v_H = 21 \frac{\text{m}}{\text{s}} \times \cos 55^\circ = \mathbf{12 \frac{\text{m}}{\text{s}}}$
- $v_V = 21 \frac{\text{m}}{\text{s}} \times \sin 55^\circ = \mathbf{17 \frac{\text{m}}{\text{s}}}$

Exercise 3.6 (b)

3.6 A player kicks a ball from level ground with a velocity of $21 \frac{\text{m}}{\text{s}}$ at 55° to the horizontal. Air resistance is negligible.

(b) **Show that** the ball reaches its maximum height at time $t = 1.75 \text{ s}$. [1]

Exercise 3.6 (b)

3.6 A player kicks a ball from level ground with a velocity of $21 \frac{\text{m}}{\text{s}}$ at 55° to the horizontal. Air resistance is negligible.

(b) **Show that** the ball reaches its maximum height at time $t = 1.75 \text{ s}$. [1]

(b) at the top, $v_V = 0$, so $v = u + at$ gives $t = u_V/g$

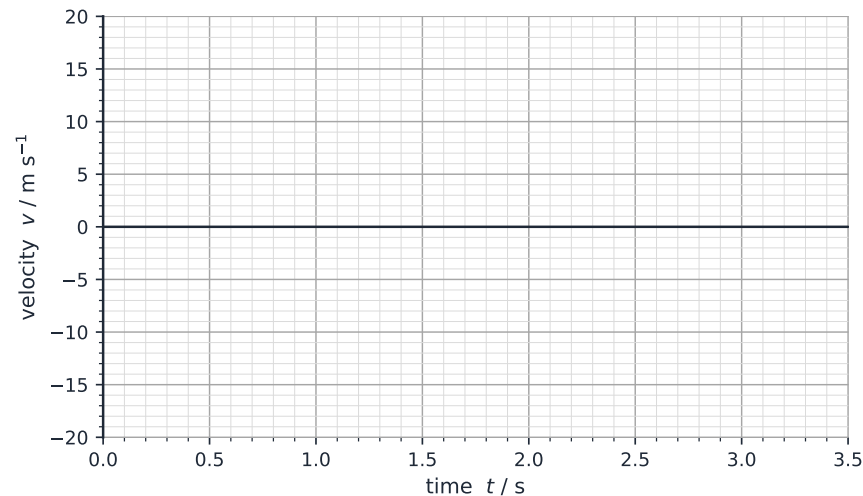
$$t = \frac{u_V}{g} = \frac{17.2 \frac{\text{m}}{\text{s}}}{9.81 \text{ m s}^{-2}} = \mathbf{1.75 \text{ s}}$$

Setup for exercise 3.6

3.6 A player kicks a ball from level ground with a velocity of $21 \frac{\text{m}}{\text{s}}$ at 55° to the horizontal. Air resistance is negligible.

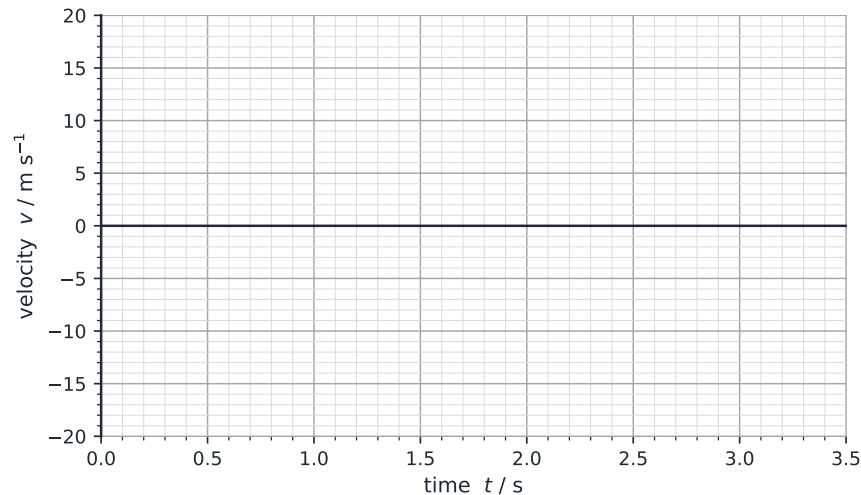
Exercise 3.6 (c)

(c) The ball lands at time $t = 3.5$ s. On the grid below, **sketch** the variation with time t of v_H and of v_V , from $t = 0$ to $t = 3.5$ s. Take the upward direction as positive. **Label** your lines H and V. [4]



Exercise 3.6 (c)

(c) The ball lands at time $t = 3.5$ s. On the grid below, **sketch** the variation with time t of v_H and of v_V , from $t = 0$ to $t = 3.5$ s. Take the upward direction as positive. **Label** your lines H and V. [4]



(c)

- **H**: horizontal straight line at $v = 12 \frac{\text{m}}{\text{s}}$ across the whole grid, labelled H

Full solution 3.6 (c)

(c)

- **H**: horizontal straight line at $v = 12 \frac{\text{m}}{\text{s}}$ across the whole grid, labelled H
- **V**: straight line of negative gradient, labelled V
- starts at $+17 \frac{\text{m}}{\text{s}}$ and ends at $-17 \frac{\text{m}}{\text{s}}$
- crosses $v = 0$ at $t = 1.75 \text{ s}$

Exercise 3.6 (d)

3.6 A player kicks a ball from level ground with a velocity of $21 \frac{\text{m}}{\text{s}}$ at 55° to the horizontal. Air resistance is negligible.

(d) **Determine** the magnitude and the direction of the velocity of the ball just before it lands. [3]

Exercise 3.6 (d)

3.6 A player kicks a ball from level ground with a velocity of $21 \frac{\text{m}}{\text{s}}$ at 55° to the horizontal. Air resistance is negligible.

(d) **Determine** the magnitude and the direction of the velocity of the ball just before it lands. [3]

(d) just before landing, $v_H = 12 \frac{\text{m}}{\text{s}}$ and $v_V = -17 \frac{\text{m}}{\text{s}}$

$$v = \sqrt{v_H^2 + v_V^2} = \sqrt{\left(12 \frac{\text{m}}{\text{s}}\right)^2 + \left(17 \frac{\text{m}}{\text{s}}\right)^2} = \mathbf{21 \frac{m}{s}}$$

$$\tan \theta = \frac{v_V}{v_H} = \frac{17 \frac{\text{m}}{\text{s}}}{12 \frac{\text{m}}{\text{s}}} \Rightarrow \theta = \mathbf{55^\circ} \text{ below the horizontal}$$

Exercise 3.6 (e)

3.6 A player kicks a ball from level ground with a velocity of $21 \frac{\text{m}}{\text{s}}$ at 55° to the horizontal. Air resistance is negligible.

(e) State why the horizontal component of the velocity does not change during the flight.[1]

Exercise 3.6 (e)

3.6 A player kicks a ball from level ground with a velocity of $21 \frac{\text{m}}{\text{s}}$ at 55° to the horizontal. Air resistance is negligible.

(e) State why the horizontal component of the velocity does not change during the flight.[1]

(e) air resistance is negligible, so there is **no horizontal force** and therefore no horizontal acceleration