

Exercise walk-through

1.3

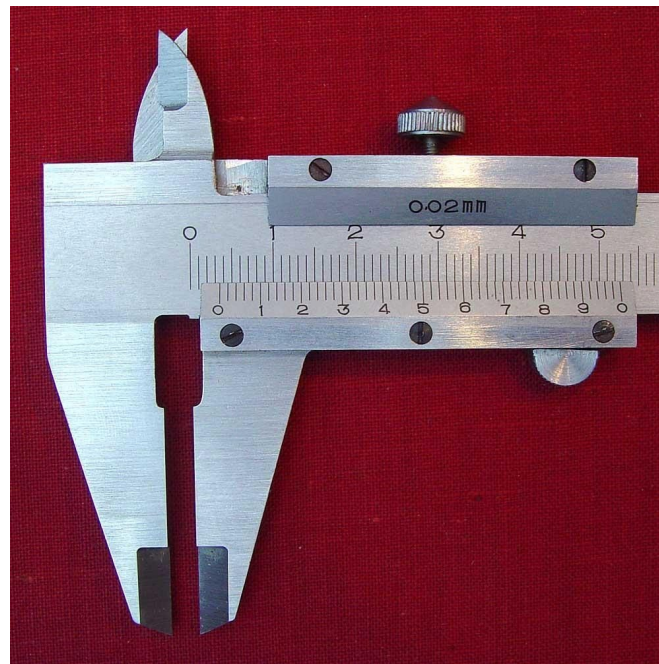
Errors and uncertainties

A-Level Physics

Every measurement is uncertain

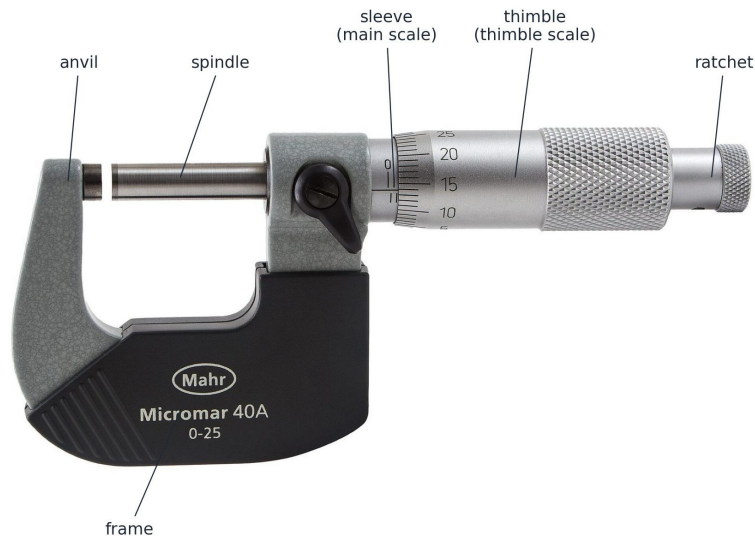
Every measurement carries some **uncertainty**. A good experimenter:

- knows where it comes from
- estimates it fairly
- carries it to the answer

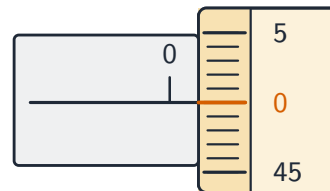


Calipers measure with a small uncertainty

The micrometer – to 0.01 mm

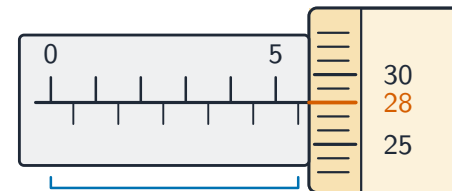


(a) zero check



reads 0.00 mm

(b) a measurement



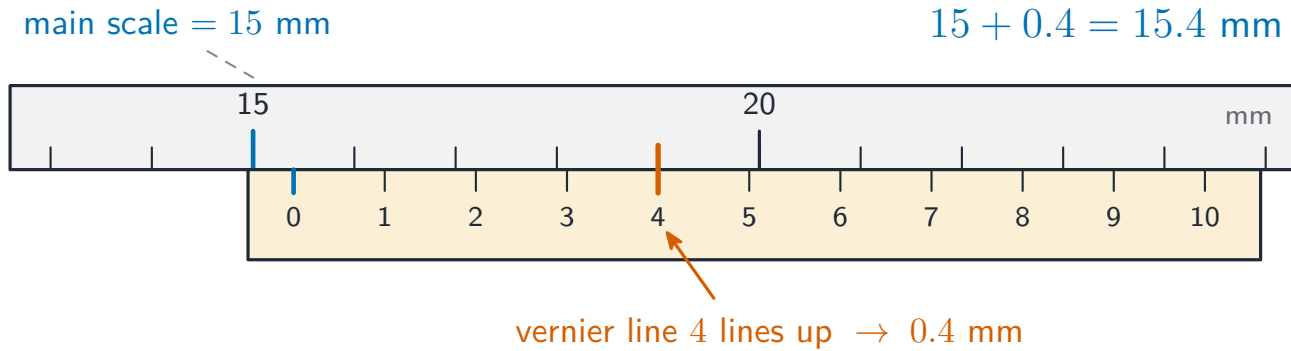
5.5 mm

$$5.5 + 0.28 = 5.78 \text{ mm}$$

A micrometer
thimble scale.

reads to 0.01 mm: reading = **sleeve** (main scale) +

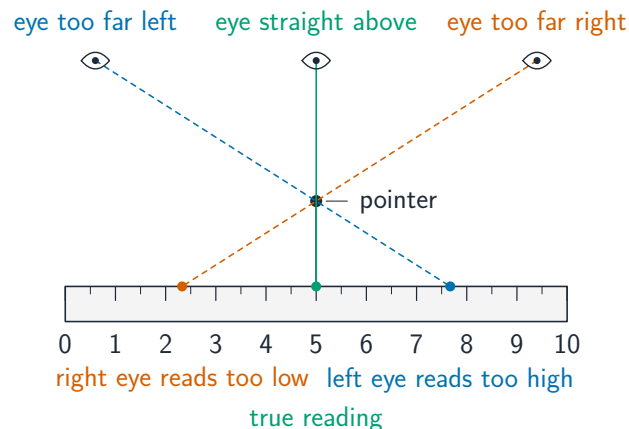
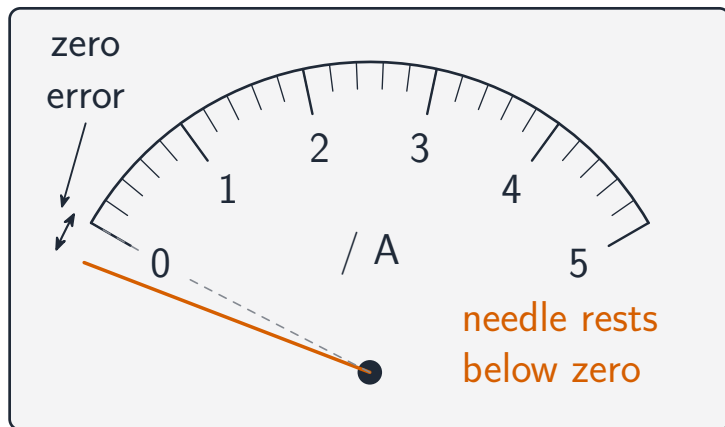
The vernier caliper – to 0.1 mm



Whole mm from the **main scale**, then tenths from the **vernier** mark that lines up.

Systematic errors

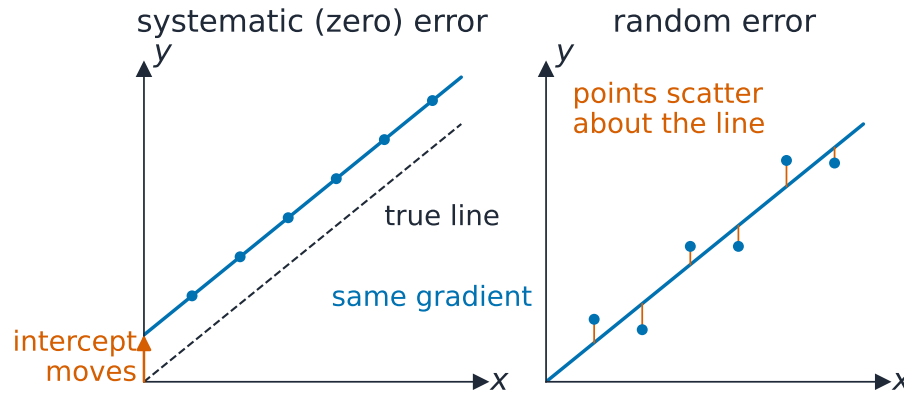
A **systematic error** shifts **every** reading by the same amount, in the same direction. Causes: a **zero error**, a **calibration** error, **parallax**.



It spoils **accuracy**, not precision, and repeating cannot reveal it. **Reduce it**: subtract the zero error; calibrate against a known standard; read the scale from directly in front.

Random errors

A **random error** scatters readings **above and below** the true value at random. On a graph it scatters the points; a systematic error moves the **intercept**.



It spoils the **precision**. **Causes:** how you read the scale, changing conditions, the smallest step the **instrument** shows. **Reduce it:** take the **mean** of many repeats; use smaller divisions; time many oscillations, not one.

Exercises 2.1-2.2

2.1 State the difference between a **systematic** error and a **random** error.

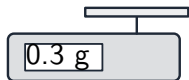
[2]

systematic error

random error

2.2 A balance reads 0.3 g with **nothing** on it. Name this error and state how to correct a reading.

[2]



Exercises 2.1-2.2

2.1 State the difference between a **systematic** error and a **random** error. [2]

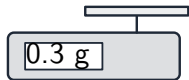
systematic error

random error

2.1

- a **systematic** error shifts every reading the **same way**; repeating does not remove it
 - a **random** error scatters readings **both ways**; the **mean** of repeats reduces it
-

2.2 A balance reads 0.3 g with **nothing** on it. Name this error and state how to correct a reading. [2]



2.2

- the balance reads a mass with nothing on it, so this is a **zero error** (systematic)
- **subtract** 0.3 g from every reading

Exercises 2.3-2.4

2.3 A length is (2.50 ± 0.05) m. Find its **percentage uncertainty**. [1]

(2.50 ± 0.05) m

2.4 State the difference between **precision** and **accuracy**. [2]

precision

accuracy

Exercises 2.3·2.4

2.3 A length is (2.50 ± 0.05) m. Find its **percentage uncertainty**. [1]

(2.50 ± 0.05) m

2.3

- percentage uncertainty = $(\Delta x/|x|) \times 100\%$

$$\%L = \frac{\Delta L}{L} \times 100\% = \frac{0.05 \text{ m}}{2.50 \text{ m}} \times 100\% = \mathbf{2\%}$$

2.4 State the difference between **precision** and **accuracy**. [2]

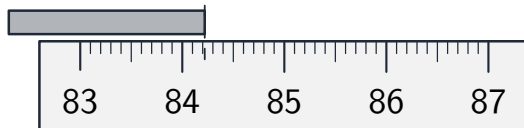
precision accuracy

2.4

- precision**: how close repeats are to **each other**
- accuracy**: how close a reading (or the mean) is to the **true value**

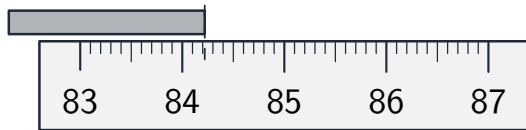
Exercise 2.5

2.5 A student measures a length with a metre rule as 84.2 cm. **Estimate** the absolute uncertainty in this reading, and explain your choice. [2]



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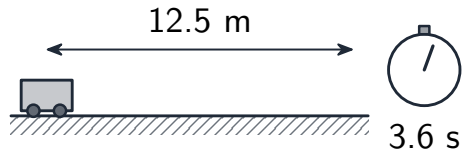


2.5

- a metre rule is marked in millimetres, so one end is ± 0.05 cm
- a length uses **two** ends, so $\Delta L = \pm 0.1$ cm is also accepted if that is explained

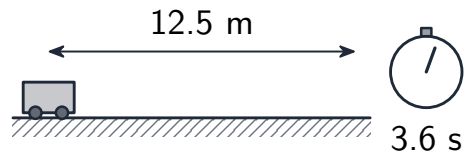
Exercise 2.6

2.6 A speed is calculated as $3.472 \frac{\text{m}}{\text{s}}$ from a distance of 12.5 m and a time of 3.6 s. State how many significant figures the answer should be given to, and **justify** your answer. [2]



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2.6

- 3.6 s is given to **2 s.f.**; 12.5 m is 3 s.f.
- a calculated answer cannot be more precise than the least precise input, so quote **2 s.f.**: $3.5 \frac{\text{m}}{\text{s}}$

Exercise 3.1

3.1 Repeated readings are **close together** but **far from** the true value. They are: [1]

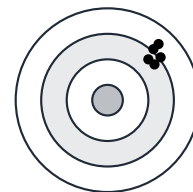
☐

A accurate but not precise

B precise but not accurate

C both accurate and precise

D neither



Exercise 3.1

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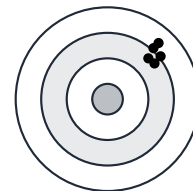
☐

A accurate but not precise

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3.1 B

- repeats that are **close together** are **precise**
- they sit **far from** the true value, so they are **not accurate**
- that pattern is a **systematic** error (e.g. a zero error)
- A swaps the two words

Exercise 3.2

3.2 Two lengths are $a = (40 \pm 1)$ mm and $b = (25 \pm 1)$ mm. **Determine** $a - b$ with its absolute uncertainty. [2]

 $a = (40 \pm 1)$ mm

 $b = (25 \pm 1)$ mm

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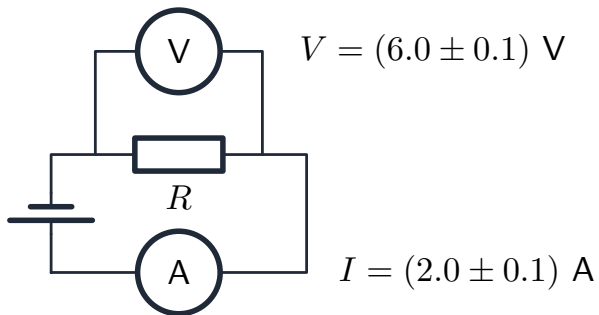
 $b = (25 \pm 1)$ mm

3.2

- subtract the lengths: $a - b = 40 \text{ mm} - 25 \text{ mm} = 15 \text{ mm}$
- for a difference, add the **absolute** uncertainties: $\Delta = \Delta a + \Delta b = 1 \text{ mm} + 1 \text{ mm} = 2 \text{ mm}$
- **(15 ± 2) mm**

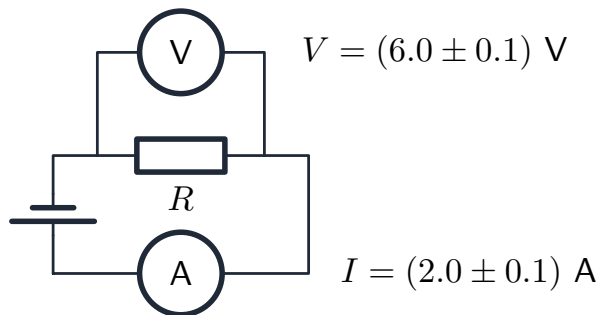
Exercise 3.3

3.3 A resistance is found from $R = V/I$, with $V = (6.0 \pm 0.1) \text{ V}$ and $I = (2.0 \pm 0.1) \text{ A}$. Find R and its **absolute uncertainty**. [4]



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3.3

- $R = V/I$

Full solution 3.3

3.3

- $R = V/I$

$$R = \frac{V}{I} = \frac{6.0 \text{ V}}{2.0 \text{ A}} = 3.0 \text{ } \Omega$$

- for a quotient, add the **percentage** uncertainties, each one $\frac{\Delta X}{X} \times 100\%$

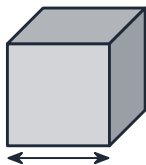
$$\%V = \frac{0.1 \text{ V}}{6.0 \text{ V}} \times 100\% = 1.7\%, \quad \%I = \frac{0.1 \text{ A}}{2.0 \text{ A}} \times 100\% = 5.0\%$$

- $\%R = 1.7\% + 5.0\% = 6.7\%$

$$\Delta R = 0.067 \times 3.0 \text{ } \Omega = 0.2 \text{ } \Omega \Rightarrow R = (\mathbf{3.0 \pm 0.2}) \text{ } \Omega$$

Exercise 3.4

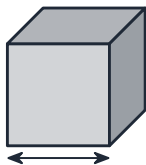
3.4 A cube has side $L = (2.00 \pm 0.02)$ cm. Its volume is $V = L^3$. Find the **percentage uncertainty** in V . [2]



$$L = (2.00 \pm 0.02) \text{ cm}$$

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$$L = (2.00 \pm 0.02) \text{ cm}$$

3.4

$$\%L = \frac{\Delta L}{L} \times 100\% = \frac{0.02 \text{ cm}}{2.00 \text{ cm}} \times 100\% = 1\%$$

- $V = L^3$, so multiply the percentage by 3: $\%V = 3 \times 1\% = \mathbf{3\%}$

Exercise 3.5 (a)

3.5 Two samples of metal, A and B, are tested to find the density of the material each is made from.

(a) **Explain** what is meant by the **accuracy** of a measured value.

[1]

Exercise 3.5 (a)

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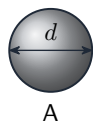
- (a) **Explain** what is meant by the **accuracy** of a measured value. [1]
- (a) **accuracy** is how close the measured value is to the **true value**

Exercise 3.5 (b)

3.5 Two samples of metal, A and B, are tested to find the density of the material each is made from.

(b) Sample A is a sphere. Its diameter is $d = (2.40 \pm 0.02)$ cm and its mass is $m = (68.5 \pm 0.5)$ g. The volume of a sphere of diameter d is $V = \frac{\pi d^3}{6}$.

Show that the density of the material of sample A is about $9.5 \times 10^3 \text{ kg m}^{-3}$. [3]

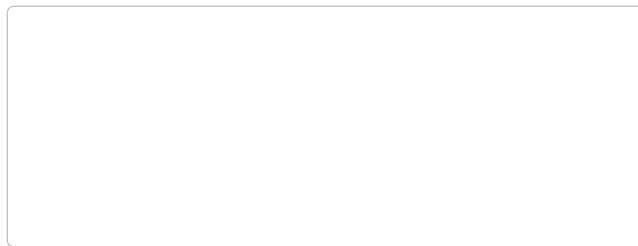


$$d = (2.40 \pm 0.02) \text{ cm}$$

$$m = (68.5 \pm 0.5) \text{ g}$$



B

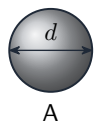


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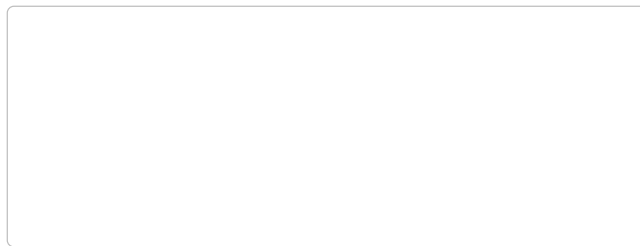


$$d = (2.40 \pm 0.02) \text{ cm}$$

$$m = (68.5 \pm 0.5) \text{ g}$$



B



(b) convert **both** quantities to SI first: $d = 2.40 \text{ cm} = 2.40 \times 10^{-2} \text{ m}$, $m = 68.5 \text{ g} = 68.5 \times 10^{-3} \text{ kg}$

$$V = \frac{\pi d^3}{6} = \frac{\pi (2.40 \times 10^{-2} \text{ m})^3}{6} = 7.24 \times 10^{-6} \text{ m}^3$$

$$\rho = \frac{m}{V} = \frac{68.5 \times 10^{-3} \text{ kg}}{7.24 \times 10^{-6} \text{ m}^3} = 9.46 \times 10^3 \text{ kg m}^{-3} \approx \mathbf{9.5 \times 10^3 \text{ kg m}^{-3}}$$

Exercise 3.5 (c)

3.5 Two samples of metal, A and B, are tested to find the density of the material each is made from.

(c) **Justify** the number of significant figures you have given for the density in (b). [1]

Exercise 3.5 (c)

3.5 Two samples of metal, A and B, are tested to find the density of the material each is made from.

(c) **Justify** the number of significant figures you have given for the density in (b). [1]

(c) both d and m are given to **3 s.f.**, so ρ cannot be quoted more precisely than **3 s.f.**

Exercise 3.5 (d)

3.5 Two samples of metal, A and B, are tested to find the density of the material each is made from.

(d) **Calculate** the percentage uncertainty in the density of the material of sample A. [3]

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(d)

$$\%m = \frac{\Delta m}{m} \times 100\% = \frac{0.5 \text{ g}}{68.5 \text{ g}} \times 100\% = 0.73\%$$

$$\%d = \frac{\Delta d}{d} \times 100\% = \frac{0.02 \text{ cm}}{2.40 \text{ cm}} \times 100\% = 0.83\%$$

- $V \propto d^3$, so $\%V = 3 \times 0.83\% = 2.5\%$
- $\%\rho = \%m + \%V = 0.73\% + 2.5\% = \mathbf{3.2\%}$

Exercise 3.5 (e)

3.5 Two samples of metal, A and B, are tested to find the density of the material each is made from.

(e) **Determine** the absolute uncertainty in the density of sample A, and write the density as $\rho \pm \Delta\rho$. [2]

Exercise 3.5 (e)

3.5 Two samples of metal, A and B, are tested to find the density of the material each is made from.

(e) **Determine** the absolute uncertainty in the density of sample A, and write the density as $\rho \pm \Delta\rho$. [2]

(e)

$$\Delta\rho = 0.032 \times 9.46 \times 10^3 \text{ kg m}^{-3} = 0.30 \times 10^3 \text{ kg m}^{-3}$$

- $\rho = (9.5 \pm 0.3) \times 10^3 \text{ kg m}^{-3}$

Exercise 3.5 (f)

3.5 Two samples of metal, A and B, are tested to find the density of the material each is made from.

(f) The density of the material of sample B is measured as $9.0 \times 10^3 \text{ kg m}^{-3} \pm 3\%$. **State and explain** whether A and B could be made from the same material. [2]

Exercise 3.5 (f)

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(f) The density of the material of sample B is measured as $9.0 \times 10^3 \text{ kg m}^{-3} \pm 3\%$. **State and explain** whether A and B could be made from the same material. [2]

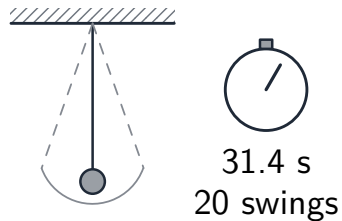
(f) write both as ranges

- A: $9.46 \times 10^3 \pm 3.2\% \Rightarrow 9.16 \times 10^3 \text{ to } 9.76 \times 10^3 \text{ kg m}^{-3}$
- B: $9.0 \times 10^3 \pm 3\% \Rightarrow 8.73 \times 10^3 \text{ to } 9.27 \times 10^3 \text{ kg m}^{-3}$
- the ranges **overlap**, so A and B **could** be the same material

Exercise 3.6 (a)

3.6 A student times **20** complete swings of a pendulum with a stopwatch and records 31.4 s. Her reaction time makes each timing uncertain by about 0.2 s.

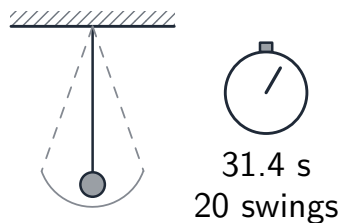
(a) **Estimate** the percentage uncertainty in her value for the **period** of one swing. [2]



Exercise 3.6 (a)

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(a) **Estimate** the percentage uncertainty in her value for the **period** of one swing. [2]



(a) the 0.2 s is the uncertainty in the **total** time, not in one swing

$$\%T = \frac{\Delta t}{t} \times 100\% = \frac{0.2 \text{ s}}{31.4 \text{ s}} \times 100\% = \mathbf{0.6\%}$$

Exercise 3.6 (b)

3.6 A student times **20** complete swings of a pendulum with a stopwatch and records 31.4 s. Her reaction time makes each timing uncertain by about 0.2 s.

(b) Explain why timing 20 swings gives a smaller percentage uncertainty than timing one. [2]

Exercise 3.6 (b)

3.6 A student times **20** complete swings of a pendulum with a stopwatch and records 31.4 s. Her reaction time makes each timing uncertain by about 0.2 s.

(b) Explain why timing 20 swings gives a smaller percentage uncertainty than timing one. [2]

(b) the absolute uncertainty stays ≈ 0.2 s (reaction time) however many swings you time

- 20 swings make the **measured time 20 times larger**, so the same Δt is a much smaller **fraction** of it