1 The random variables X and Y have independent distributions $X \sim \text{Po}(3)$ and $Y \sim \text{Po}(2)$ respectively.

(a) Find $\text{P}(2 < X < 5)$. [2]

(b) Find $\text{P}(X + Y > 2)$. [3]

(c) The total of 100 random values of X and 150 random values of Y is denoted by T.

Use a suitable approximating distribution to find $P(T < 560)$. [4]

2 The mean mass of packets of Trueleaf tea is supposed to be 500 grams. An inspector wishes to test whether this value is correct. He weighs 60 randomly chosen packets and notes the mass, x grams, of each packet. The results are summarised as follows.

$$n = 60 \qquad \sum x = 29\,970 \qquad \sum x^2 = 14\,970\,300$$

Test, at the 5% significance level, whether the population mean mass is 500 grams. [8]

4 The masses of a certain species of animal are known to be normally distributed with standard deviation σ kg. A researcher obtains the masses of a random sample of n animals of this species and uses these masses to find two confidence intervals (α% and 90%) for the population mean. The width of the α% confidence interval is $1.414 \times$ the width of the 90% confidence interval.

(a) Find α. [3]

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(b) Find the probability that the 90% confidence interval contains the population mean given that the α% confidence interval contains the population mean. [1]

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6 The masses, in kilograms, of large and small bags of potatoes have the independent distributions N(2.5, 0.05) and N(0.8, 0.02) respectively.

(a) Find the probability that the total mass of a randomly chosen large bag of potatoes and a randomly chosen small bag of potatoes is more than 3.55 kg. [5]

(b) Find the probability that the mass of a randomly chosen large bag of potatoes is less than 3 times the mass of a randomly chosen small bag of potatoes. [5]

7 The time, in minutes, taken by students to complete a test is modelled by the random variable X with probability density function

$$\mathrm{f}(x) = \begin{cases} -\frac{3}{4}(x-3)(x-5) & 3 \leqslant x \leqslant 5, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the probability that a randomly chosen student takes longer than 4.5 minutes to complete the test. [4]

(b) Write down the median of X. [1]

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(c) Without performing an integration, use your answer to part **(a)** to find $P(3.5 < X < 4.5)$. [2]

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Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.