

Probability & Statistics 2

A-Level Mathematics • Topic 6

A-Level Mathematics



Probability & Statistics 2

What we will cover

- 1 The Poisson distribution
- 2 Combining random variables
- 3 Continuous random variables
- 4 Sampling & estimation
- 5 Hypothesis tests
- 6 Recap



1 Poisson and combinations

Probability & Statistics 2

└ Poisson and combinations

The Poisson distribution

Counts of rare, independent events in a fixed interval: $X \sim \text{Po}(\lambda)$,

$$P(X = k) = e^{-\lambda} \frac{\lambda^k}{k!}, \quad E(X) = \text{Var}(X) = \lambda.$$

A binomial $X \sim B(n, p)$ with n large and p small is approximately Poisson with $\lambda = np$.

Probability & Statistics 2

└ Poisson and combinations

Combining random variables

For a linear change and for independent variables:

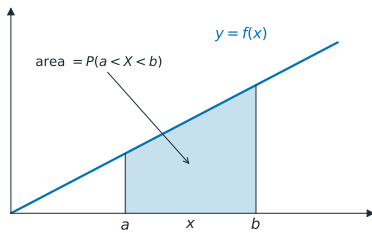
$$E(aX + b) = aE(X) + b$$

$$\text{Var}(aX + bY) = a^2 \text{Var} X + b^2 \text{Var} Y$$

e.g. mean 5, variance 4: $E(3X - 1) = 14$, $\text{Var}(3X - 1) = 36$.

2 Continuous and sampling

Continuous random variables

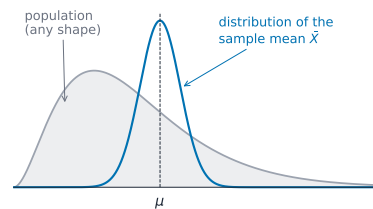


A **density function** 概率密度函数
 $f(x) \geq 0$, $\int f = 1$.

$$P(a < X < b) = \int_a^b f, \quad E(X) = \int xf.$$

Median 中位数 solves $F(m) = 0.5$.

Sampling and estimation



CLT 中心极限定理: $\bar{X}$ near-normal

$E(\bar{X}) = \mu$, $\text{Var}(\bar{X}) = \frac{\sigma^2}{n}$. A 95% **confidence interval** 置信区间:

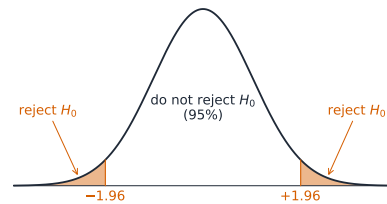
$$\bar{x} \pm 1.96 \frac{\sigma}{\sqrt{n}}$$

$$n = 64, \bar{x} = 50, \sigma = 8$$

$$50 \pm 1.96 \Rightarrow (48.0, 52.0)$$

3 Hypothesis tests

Hypothesis tests



Null 原假设 H_0 vs **alternative** 备择假设 H_1 ; pick a **significance level** 显著性水平.

Test statistic in the **rejection region** 拒绝域 $\Rightarrow$ reject H_0 .

Type I 第一类错误: reject a true H_0 ; **Type II** 第二类错误: accept a false H_0 .

Worked example – a Poisson probability

$X \sim \text{Po}(3)$. Find $P(X = 2)$.

$$P(X = 2) = e^{-3} \frac{3^2}{2!} = e^{-3} \times 4.5 = 0.224$$

For a Poisson variable the mean and variance both equal λ . Use $P(X = r) = e^{-\lambda} \frac{\lambda^r}{r!}$.

4 Recap

Topic 6 – key takeaways

1 Poisson

$Po(\lambda)$; mean = variance = λ

2 Combining

Var adds for independent variables

3 Sampling

CLT; $\bar{x} \pm 1.96\sigma/\sqrt{n}$

4 Testing

H_0 vs H_1 ; reject in the tail

Check yourself

- 1 Mean and variance of $Po(\lambda)$?
- 2 $\text{Var}(2X)$ if $\text{Var}(X) = 5$?
- 3 What does the area under $f(x)$ give?
- 4 Rejecting a true H_0 is which error?



Answers 1. both λ 2. $4 \times 5 = 20$ 3. a probability 4. Type I