

Probability & Statistics 2

A-Level Mathematics ■ Topic 6

A-Level Mathematics



What we will cover

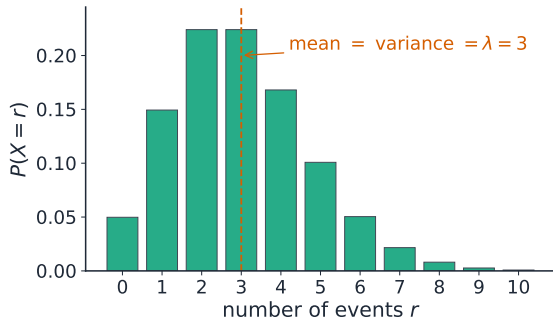
- 1 The Poisson distribution
- 2 Combining random variables
- 3 Continuous random variables
- 4 Sampling & estimation
- 5 Hypothesis tests
- 6 Recap



1

Poisson and combinations

The Poisson distribution



Poisson queue

Combining random variables

For a linear change and for **independent** variables:

$$E(aX + b) = aE(X) + b$$

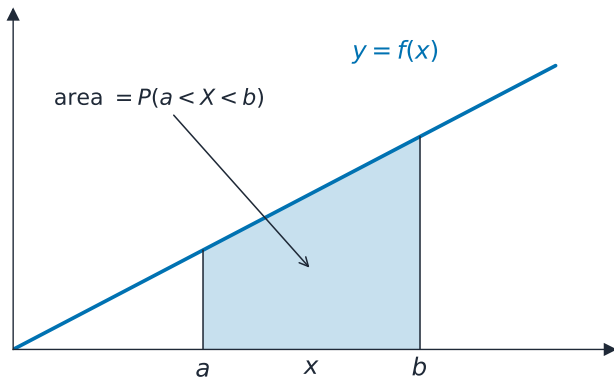
$$\text{Var}(aX + bY) = a^2 \text{Var} X + b^2 \text{Var} Y$$

e.g. mean 5, variance 4: $E(3X - 1) = 14$, $\text{Var}(3X - 1) = 36$.

2

Continuous and sampling

Continuous random variables

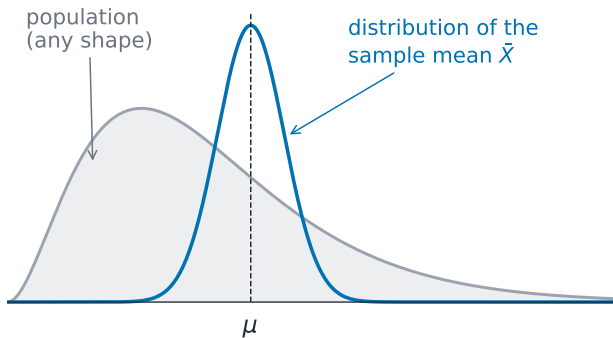


A density function 概率密度函数
 $f(x) \geq 0$, $\int f = 1$.

$$P(a < X < b) = \int_a^b f, \quad E(X) = \int xf.$$

Median 中位数 solves $F(m) = 0.5$.

Sampling and estimation



CLT 中心极限定理: $\bar{X}$ near-normal

$E(\bar{X}) = \mu, \text{Var}(\bar{X}) = \frac{\sigma^2}{n}$. A 95%
confidence interval 置信区间:

$$\bar{x} \pm 1.96 \frac{\sigma}{\sqrt{n}}.$$

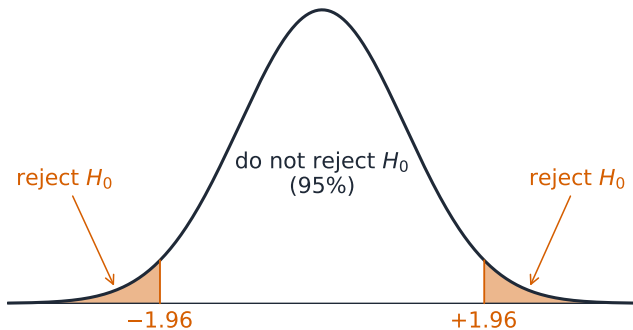
$$n = 64, \bar{x} = 50, \sigma = 8$$

$$50 \pm 1.96 \Rightarrow (48.0, 52.0)$$

3

Hypothesis tests

Hypothesis tests



Null 原假设 H_0 vs
alternative 备择假设 H_1 ; pick a
significance level 显著性水平.

Test statistic in the **rejection region** 拒绝域 $\Rightarrow$ reject H_0 .

Type I 第一类错误: reject a true H_0 ; **Type II** 第二类错误: accept a false H_0 .

Worked example – a Poisson probability

$X \sim \text{Po}(3)$. Find $P(X = 2)$.

$$P(X = 2) = e^{-3} \frac{3^2}{2!} = e^{-3} \times 4.5 = 0.224$$

For a Poisson variable the mean **and** variance both equal λ . Use $P(X = r) = e^{-\lambda} \frac{\lambda^r}{r!}$.

4

Recap

Topic 6 – key takeaways

1

Poisson

$\text{Po}(\lambda)$; mean = variance = λ

2

Combining

Var adds for independent variables

3

Sampling

CLT; $\bar{x} \pm 1.96\sigma/\sqrt{n}$

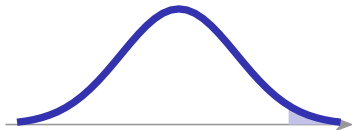
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Testing

H_0 vs H_1 ; reject in the tail

Check yourself

- 1 Mean and variance of $Po(\lambda)$?
- 2 $\text{Var}(2X)$ if $\text{Var}(X) = 5$?
- 3 What does the area under $f(x)$ give?
- 4 Rejecting a true H_0 is which error?



Answers 1. both λ 2. $4 \times 5 = 20$ 3. a probability 4. Type I