

Probability & Statistics 1

A-Level Mathematics ■ Topic 5

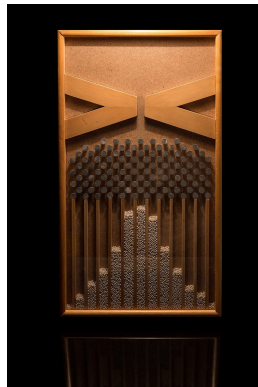
A-Level Mathematics



Probability & Statistics 1

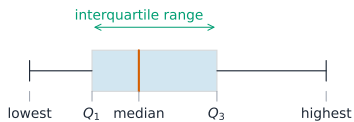
What we will cover

- 1 Representing data
- 2 Permutations & combinations
- 3 Probability
- 4 Discrete random variables
- 5 The normal distribution
- 6 Recap



1 Data

Representing data



box plot: quartiles



Dice

Also: stem-and-leaf, **histograms** 直方图 (bar area = frequency).

Averages and spread

- **mean** 平均数 $\bar{x} = \frac{\sum x}{n}$, **median** 中位数, **mode** 众数
- **standard deviation** 标准差

$$\sigma = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2}$$
- **interquartile range** 四分位距 $Q_3 - Q_1$

$$n = 10, \sum x = 50, \sum x^2 = 300$$

$$\bar{x} = 5, \sigma = \sqrt{30 - 25} = 2.24$$

2

Counting and probability

Permutations and combinations

Permutation 排列 (order matters): ${}^n P_r = \frac{n!}{(n-r)!}$.

Combination 组合: ${}^n C_r = \binom{n}{r}$.

NEEDLESS (E×3, S×2)

$$\frac{8!}{3!2!} = 3360$$

permutation (order matters) **combination (order does not)**

AB

BA

{A, B}

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

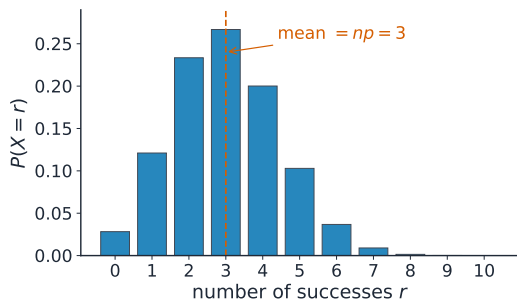
Independent: $P(A \cap B) = P(A)P(B)$. Conditional: $P(A | B) = P(A \cap B)/P(B)$.

Mutually exclusive $\Rightarrow$ intersection is empty, so you just add. Independent is not the same as exclusive.

3

Distributions

Discrete random variables



$$E(X) = \sum x P(X=x).$$

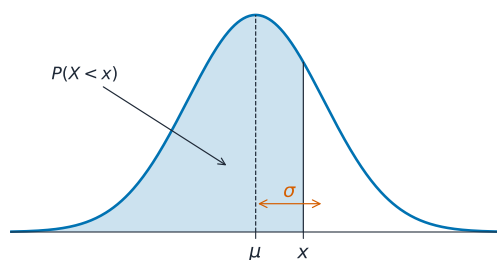
Binomial 二项分布 $B(n, p)$:

$$P(X=r) = \binom{n}{r} p^r (1-p)^{n-r}, \quad E(X) = np.$$

$$B(10, 0.3)$$

$$P(X=2) = 0.233, \quad E(X) = 3$$

The normal distribution



Normal 正态分布 $X \sim N(\mu, \sigma^2)$.

Standardise 标准化:

$$Z = \frac{X - \mu}{\sigma} \sim N(0, 1).$$

Read $P(Z < z)$ from the Φ table; a large- n binomial $\approx$ normal (with a **continuity correction** 连续性校正).

Worked example – mean and standard deviation

For 10 values, $\sum x = 50$ and $\sum x^2 = 300$.
Find the mean and standard deviation.

$$\bar{x} = \frac{50}{10} = 5$$

$$\sigma = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} = \sqrt{30 - 25} = 2.24$$

Variance = $\frac{\sum x^2}{n} - \bar{x}^2$ – the mean of the squares minus the square of the mean. Subtract in **that** order.

4 Recap

Topic 5 – key takeaways

1 Data

mean, median, mode; standard deviation

2 Counting

nP_r order matters; nC_r not

3 Probability

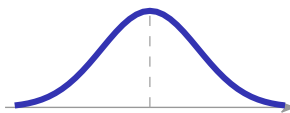
add for “or”, multiply for independent “and”

4 Distributions

binomial $B(n, p)$; normal $N(\mu, \sigma^2)$

Check yourself

- 1 Does order matter for a combination?
- 2 The test for independence of A and B ?
- 3 $E(X)$ for $X \sim B(20, 0.5)$?
- 4 How do you standardise a normal variable?



Answers 1. no 2. $P(A \cap B) = P(A)P(B)$ 3. $np = 10$ 4. $Z = (X - \mu)/\sigma$