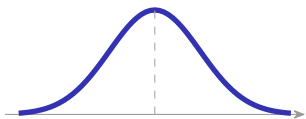
Probability & Statistics 1

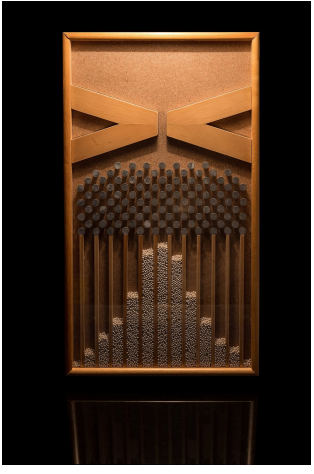
A-Level Mathematics ▪ Topic 5

A-Level Mathematics



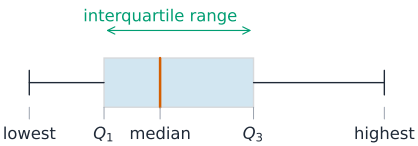
What we will cover

1. Representing data
2. Permutations & combinations
3. Probability
4. Discrete random variables
5. The normal distribution
6. Recap



1 Data

Representing data



box plot: quartiles

Dice

Also: stem-and-leaf, **histograms** 直方图 (bar *area* = frequency).

Averages and spread

- **mean** 平均数 $\bar{x} = \frac{\sum x}{n}$, **median** 中位数, **mode** 众数
- **standard deviation** 标准差 $\sigma = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2}$
- **interquartile range** 四分位距 $Q_3 - Q_1$

$n = 10$, $\sum x = 50$, $\sum x^2 = 300$

$\bar{x} = 5$, $\sigma = \sqrt{30 - 25} = 2.24$

2 Counting and probability

Permutations and combinations

Permutation 排列 (order matters): ${}^nP_r = \frac{n!}{(n-r)!}$.

Combination 组合: ${}^nC_r = \binom{n}{r}$.

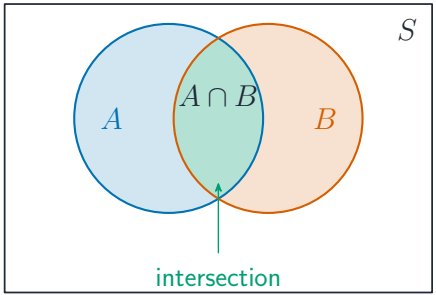
NEEDLESS (E×3, S×2)

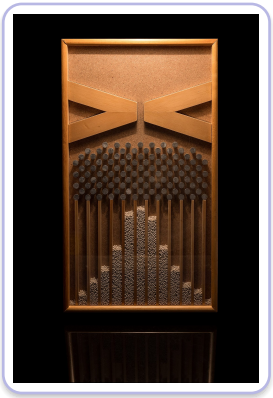
$$\frac{8!}{3!\,2!} = 3360$$

permutation (order matters) **combination (order does not)**

AB BA $\{A, B\}$

Probability

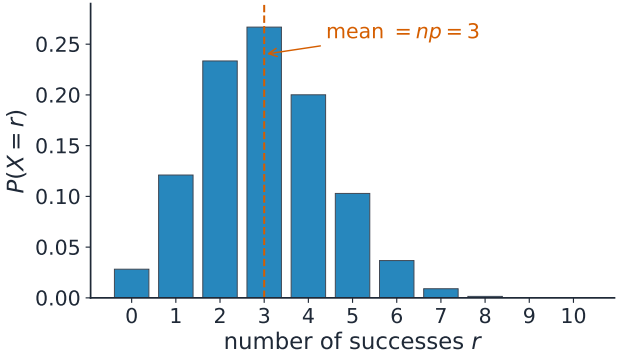




Normal galton

3 Distributions

Discrete random variables



$E(X) = \sum x\,P(X{=}x)$.

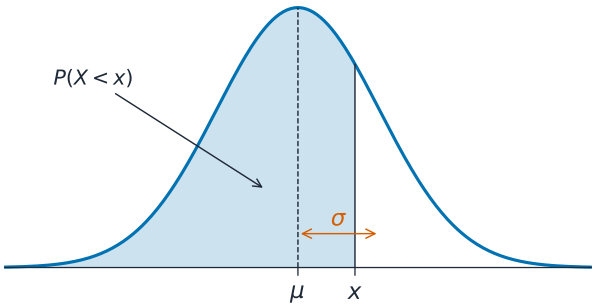
Binomial 二项分布 $B(n, p)$:

$$P(X{=}r) = \binom{n}{r} p^r (1-p)^{n-r},\ E(X) = np.$$

$B(10, 0.3)$

$P(X{=}2) = 0.233,\ E(X) = 3$

The normal distribution



Normal 正态分布 $X \sim N(\mu, \sigma^2)$.
Standardise 标准化:

$$Z = \frac{X - \mu}{\sigma} \sim N(0, 1).$$

Read $P(Z < z)$ from the Φ table; a large-n binomial $\approx$ normal (with a **continuity correction** 连续性校正).

Worked example – mean and standard deviation

For 10 values, $\sum x = 50$ and $\sum x^2 = 300$. Find the mean and standard deviation.

$$\bar{x} = \frac{50}{10} = 5$$

$$\sigma = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} = \sqrt{30 - 25} = 2.24$$

Variance $= \dfrac{\sum x^2}{n} - \bar{x}^2$ – the mean of the squares minus the square of the mean. Subtract in that order.

4 Recap

Topic 5 – key takeaways

1 Data

mean, median, mode; standard deviation

2 Counting

${}^nP_r$ order matters; ${}^nC_r$ not

3 Probability

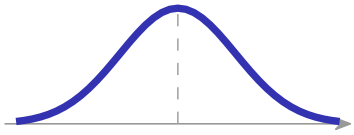
add for "or", multiply for independent "and"

4 Distributions

binomial $B(n, p)$; normal $N(\mu, \sigma^2)$

Check yourself

1. Does order matter for a combination?
2. The test for independence of A and B?
3. $E(X)$ for $X \sim B(20, 0.5)$?
4. How do you standardise a normal variable?



Answers **1.** no **2.** $P(A \cap B) = P(A)P(B)$ **3.** $np = 10$ **4.** $Z = (X - \mu)/\sigma$