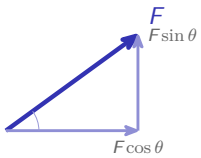
Mechanics

A-Level Mathematics ▪ Topic 4

A-Level Mathematics



What we will cover

1. Forces & equilibrium
2. Kinematics
3. Momentum
4. Newton's laws
5. Energy work & power
6. Recap

1 Forces

Forces and equilibrium

Resolve every force into perpendicular components. At equilibrium the vector sum is zero:

$$\sum F_x = 0, \qquad \sum F_y = 0.$$

A particle in equilibrium can be drawn as a closed vector triangle.

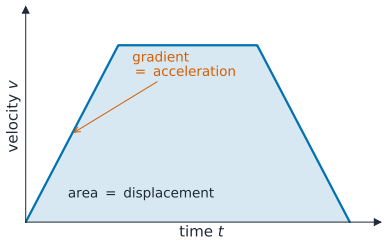
Friction

$F \leq \mu R$. Equality is limiting friction (on the point of slipping). μ is the coefficient of friction; R is the normal reaction.

> Friction always opposes the (potential) slip. If the object is at rest and not about to move, F can be less than μR.

2 Kinematics and momentum

Kinematics



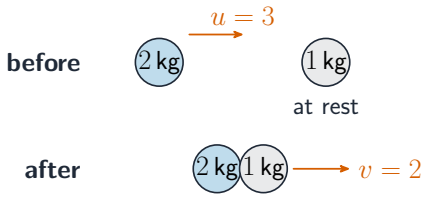
v–t graph: area = displacement, gradient = acceleration.

$v = u+at, \quad s = ut+\frac{1}{2}at^2, \quad v^2 = u^2+2$

Rest, $a = 2.5$, $t = 4\,\mathrm{s}$

$v = 10\,\mathrm{m/s}$, $s = 20\,\mathrm{m}$

Momentum



$$2(3) + 1(0) = (2+1)\,v$$

Conservation of momentum 动量守恒:

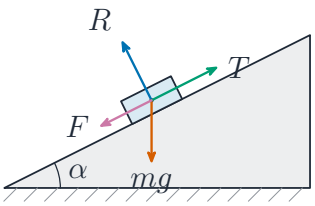
$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2.$$

$2\,\mathrm{kg}$ at 3 hits still $1\,\mathrm{kg}$, stick

$$v = \frac{6}{3} = 2\,\mathrm{m/s}$$

3 Newton and energy

Newton's laws



$F = ma, \qquad W = mg.$

On an **inclined plane** 斜面: resolve along and perpendicular to the slope, then apply $F = ma$.

Pull $12\,\mathrm{kg}$ up a $20°$ rough slope

$T \approx 110\,\mathrm{N}$

Energy, work and power

$W = Fd\cos\theta, \quad \mathrm{KE} = \frac{1}{2}mv^2, \quad \mathrm{PE} = mgh.$

No friction $\Rightarrow$ total energy conserved. Power $P = Fv$.

$12\,\mathrm{kW}$ engine at $20\,\mathrm{m/s}$

$$F = \frac{P}{v} = 600\,\mathrm{N}$$

4 Recap

Topic 4 – key takeaways

1 Forces

resolve into components; $F \leqslant \mu R$

2 Kinematics

suvat; v–t area & gradient

3 Newton

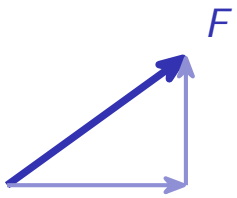
$F = ma$; resolve on a slope

4 Energy

$\frac{1}{2}mv^2$, mgh; $P = Fv$

Check yourself

1. Maximum friction before slipping?
2. What does the area under a v–t graph give?
3. Driving force of a $10\,\mathrm{kW}$ engine at $25\,\mathrm{m/s}$?
4. Quantity conserved in a collision?



Answers **1.** μR **2.** displacement **3.** $P/v = 400\,\mathrm{N}$ **4.** momentum