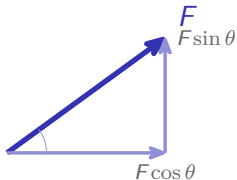


# Mechanics

## A-Level Mathematics ■ Topic 4

### A-Level Mathematics



# What we will cover

- 1 Forces & equilibrium
- 2 Kinematics
- 3 Momentum
- 4 Newton's laws
- 5 Energy work & power
- 6 Recap



1

Forces

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# Forces and equilibrium

Resolve every force into perpendicular components. At equilibrium the vector sum is zero:

$$\sum F_x = 0, \quad \sum F_y = 0.$$

A particle in equilibrium can be drawn as a closed vector triangle.

# Friction

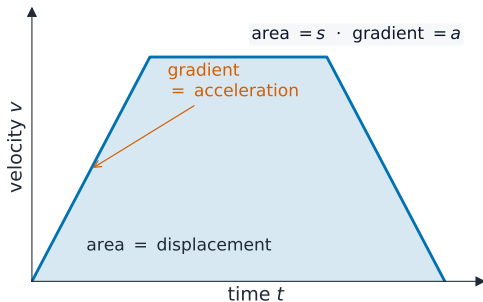
$F \leq \mu R$ . Equality is limiting friction (on the point of slipping).  $\mu$  is the coefficient of friction;  $R$  is the normal reaction.

Friction always opposes the (potential) slip. If the object is at rest and not about to move,  $F$  can be less than  $\mu R$ .

# 2

## Kinematics and momentum

# Kinematics



$v$ - $t$  graph: area = displacement,  
gradient = acceleration.

$$v = u + at \quad v^2 = u^2 + 2as$$

$$s = ut + \frac{1}{2}at^2$$

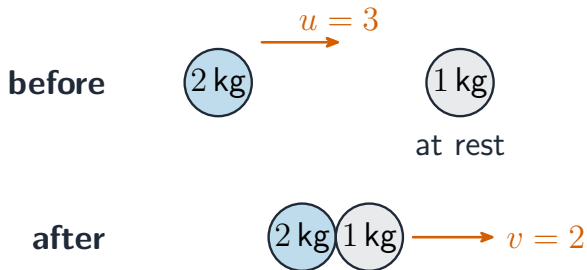
constant acceleration only

**Rest,  $a = 2.5$ ,  $t = 4$  s**

$v = 10 \text{ m/s}$ ,  $s = 20 \text{ m}$

The shaded area gives the distance travelled; the slope of the line gives the acceleration.

# Momentum



$$2(3) + 1(0) = (2 + 1)v$$

**Conservation of momentum** 动量守恒:

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

**2 kg at 3 hits still 1 kg, stick**

$$v = \frac{6}{3} = 2 \text{ m/s}$$

## Worked example – a perfectly inelastic collision

**A 2 kg body at  $3 \text{ m s}^{-1}$  hits a stationary 1 kg body and they stick together. Find the common speed.**

- **conservation of linear momentum** 动量守恒:

$$m_1 u_1 = (m_1 + m_2) v$$

- $2(3) = 3v$

- $v = 2 \text{ m s}^{-1}$

Momentum is conserved;  
kinetic energy is not.

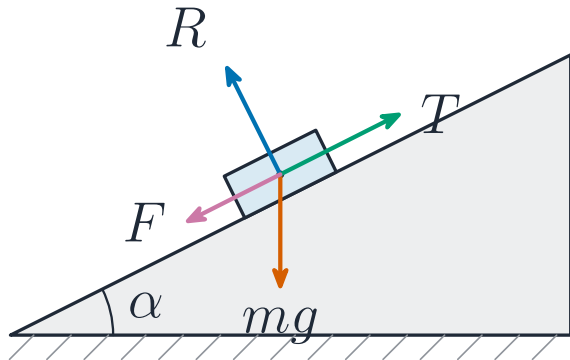
Here 9 J before becomes 6 J after – the missing 3 J went into deformation and heat.

# 3

## Newton and energy

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# Newton's laws



$$F = ma, \quad W = mg$$

On an **inclined plane** 斜面:  
resolve along and perpendicular to  
the slope, then apply  $F = ma$ .

**Pull 12 kg up a  $20^\circ$  rough  
slope**

$$T \approx 110 \text{ N}$$

# The laws, and the forces they name

## Newton's laws of motion

**Newton's laws of motion** 牛顿运动定律: a body keeps its velocity unless a **resultant** 合力 force acts;  $F = ma$ ; and every force has an equal and opposite partner.

## Newton's third law

**Newton's third law** 牛顿第三定律 pairs act on *different* bodies – which is why they never cancel in one free-body diagram.

## Connected particles

For **connected particles** 连接的物体 write  $F = ma$  for each body separately, with the same acceleration and the tension appearing twice, in opposite directions.

## Thrust

A light rod can push as well as pull: the compression force is **thrust** 推力, and a string can never provide it.

**Limiting equilibrium** 临界平衡 is the instant before sliding: friction is at its maximum  $\mu R$ , and the body is still in equilibrium.

# Energy, work and power

$$W = Fd \cos \theta, \quad \text{KE} = \frac{1}{2}mv^2, \quad \text{PE} = mgh.$$

No friction  $\Rightarrow$  total energy conserved. Power  
 $P = Fv$ .

**12 kW engine at 20 m/s**

$$F = \frac{P}{v} = 600 \text{ N}$$

# The energy quantities you can calculate

**kinetic energy**

$$\frac{1}{2}mv^2$$

**gravitational potential energy** 重力势能

$mgh$ , measured from a stated level

**work done**

$Fd$  – force times distance **along the force**

**power**

$$\frac{\text{work}}{\text{time}} = Fv \text{ for a constant force}$$

**conservation of energy** 能量守恒

with no resistance,  $KE + PE$  is constant

**with resistance**

work done against **air resistance** 空气阻力 or friction is the energy lost from that total

On a **velocity-time graph** 速度时间图 the **area** is the displacement and the **gradient** is the acceleration, with a negative gradient meaning **deceleration** 减速.

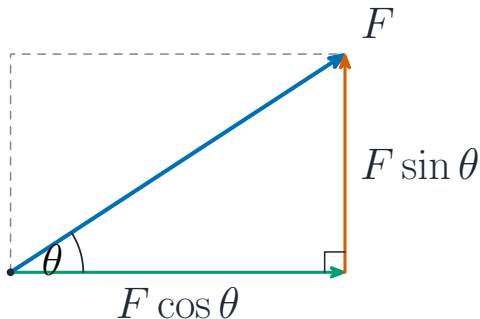
## Three forces on a climber



Every mechanics problem is a picture like this. Three forces act on the climber – weight straight down, tension along the rope, and a push from the rock – and because they balance, the climber hangs still in equilibrium. Resolving each into horizontal and vertical parts turns the picture into equations

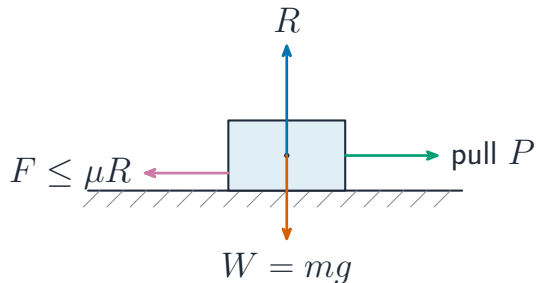
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## Forces and equilibrium, continued (2)



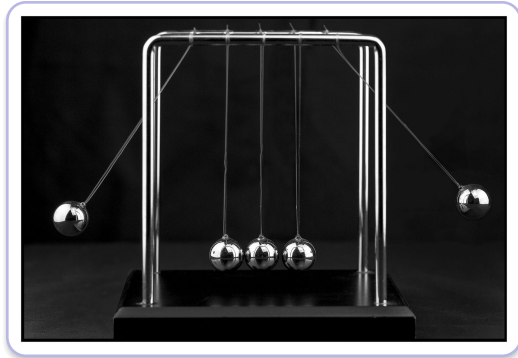
A force at angle  $\theta$  has a horizontal part  $F \cos \theta$  and a vertical part  $F \sin \theta$ .

## Normal reaction and friction on a rough surface



On a rough surface the contact force splits into the normal reaction  $R$  and friction  $F$  (at most  $\mu R$ ).

# Newton's cradle and conservation of momentum



A Newton's cradle demonstrates conservation of momentum in collisions.

## Exam tips

- Draw a clear **force diagram** and resolve into perpendicular components; for equilibrium, each direction sums to zero.
- Use **SUVAT** only for constant acceleration and keep a consistent positive direction.
- Apply  $F = ma$  along the direction of motion, including friction ( $F = \mu R$ ) on a rough surface.
- State your **assumptions** (light inextensible string, smooth pulley, particle) – they are often worth a mark.

4

Recap

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## Topic 4 – key takeaways

1

### Forces

resolve into components;  $F \leq \mu R$

2

### Kinematics

suvat;  $v$ - $t$  area & gradient

3

### Newton

$F = ma$ ; resolve on a slope

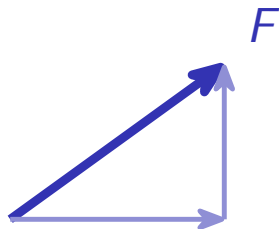
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### Energy

$\frac{1}{2}mv^2$ ,  $mgh$ ;  $P = Fv$

## Check yourself

- 1 Maximum friction before slipping?
- 2 What does the area under a  $v$ - $t$  graph give?
- 3 Driving force of a 10 kW engine at 25 m/s?
- 4 Quantity conserved in a collision?



**Answers** 1.  $\mu R$  2. displacement 3.  $P/v = 400 \text{ N}$  4. momentum