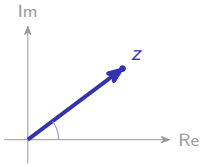
Pure Mathematics 3

A-Level Mathematics ▪ Topic 3

A-Level Mathematics



What we will cover

1 Algebra and integration

Partial fractions and integration

Split a proper rational function into simpler pieces, then integrate term by term:

$$\frac{1}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}.$$

Repeated factors need a squared denominator; a quadratic factor that will not factor stays as a linear numerator over it.

> $\int \frac{1}{x}\,dx = \ln|x| + C$. Partial fractions turn many rational integrals into logs.

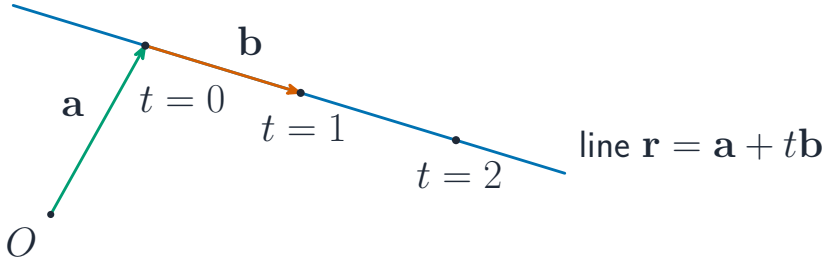
2 Vectors

Vectors

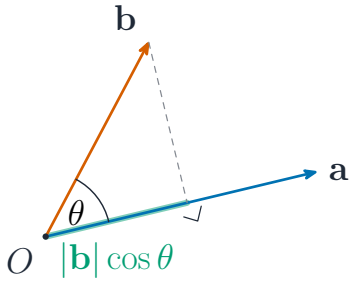
$|\mathbf{v}| = \sqrt{x^2 + y^2 + z^2}$; a **unit vector** 单位向量 has length 1.

$$\mathbf{r} = \mathbf{a} + t\mathbf{b} \text{ (a line).}$$

Lines can be parallel, intersect, or be **skew** 异面直线.



The scalar product



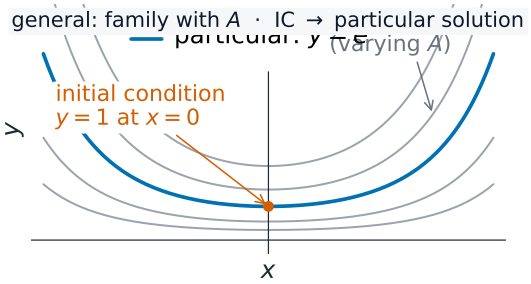
$\mathbf{a}\cdot\mathbf{b} = a_1b_1+a_2b_2+a_3b_3 = |\mathbf{a}||\mathbf{b}|\cos\theta.$

$\mathbf{a}\cdot\mathbf{b} = 0 \Rightarrow$ perpendicular.

e.g. angle between $(1,2,2)$ and $(2,2,1)$: $\cos\theta = \frac{8}{9}$, $\theta = 27.3°$.

3 Differential equations

Differential equations



Separable 可分离变量: split y and x, then integrate both sides.

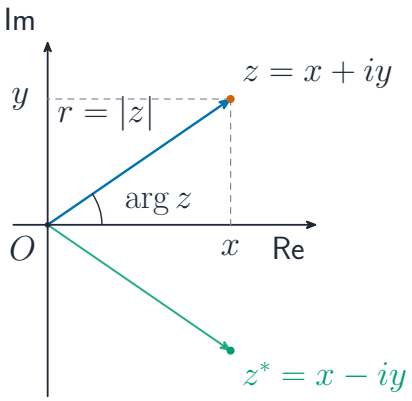
$\frac{dy}{dx} = xy$, $y = 1$ **at** $x = 0$

$\ln y = \frac{1}{2}x^2 + c \Rightarrow y = e^{x^2/2}$

A condition fixes C (the **particular solution** 特解).

4 Complex numbers

Complex numbers



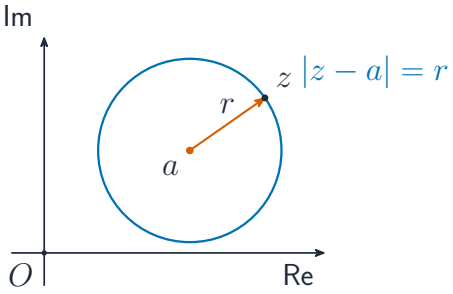
$z = x + iy$, $i^2 = -1$. **Conjugate** 共轭 $z^* = x - iy$.

$$|z| = \sqrt{x^2 + y^2}, \quad z = re^{i\theta}.$$

Divide by multiplying by the conjugate:

$$\frac{3+i}{1-i}$$
$$= 1 + 2i$$

Loci on the Argand diagram



A condition on z draws a **locus** 轨迹:

- $|z - a| = r$ – a circle
- $|z - a| = |z - b|$ – a perpendicular bisector
- $\arg(z - a) = \theta$ – a half-line

Worked example – integration by parts

Find $\int x\cos x\,dx$.

By parts with $u = x$, $\dfrac{dv}{dx} = \cos x$ (so $v = \sin x$):

$$\int x\cos x\,dx = x\sin x - \int \sin x\,dx$$
$$= x\sin x + \cos x + C$$

Choose u to be the part that gets simpler when differentiated – here $x \to 1$. The other part must be integrable.

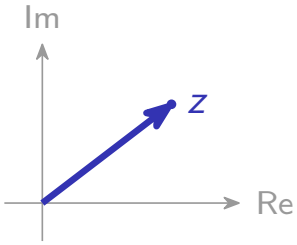
5 Recap

Topic 3 – key takeaways

1. **Integration**
   partial fractions, by parts, substitution
2. **Vectors**
   $\mathbf{r} = \mathbf{a} + t\mathbf{b}$; $\mathbf{a}\cdot\mathbf{b} = 0$ perpendicular
3. **Diff. equations**
   separate variables; a condition fixes C
4. **Complex**
   $z = x + iy = re^{i\theta}$; loci on the Argand plane

Check yourself

1. What does $\mathbf{a} \cdot \mathbf{b} = 0$ tell you?
2. Which integration method suits $\int x \cos x \, dx$?
3. The conjugate of $3 - 2i$?
4. What locus is $|z - a| = r$?



Answers **1.** they are perpendicular **2.** by parts **3.** $3+2i$ **4.** a circle, centre a, radius r