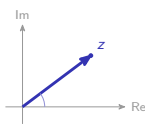


Pure Mathematics 3
A-Level Mathematics • Topic 3

A-Level Mathematics



Pure Mathematics 3

What we will cover

- 1 Partial fractions & integration
- 2 Vectors
- 3 Differential equations
- 4 Complex numbers
- 5 Recap



1 Algebra and integration

Pure Mathematics 3
Algebra and integration

Partial fractions and integration

Split a proper rational function into simpler pieces, then integrate term by term:

$$\frac{1}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}.$$

Repeated factors need a squared denominator; a quadratic factor that will not factor stays as a linear numerator over it.

$\int \frac{1}{x} dx = \ln |x| + C$. Partial fractions turn many rational integrals into logs.

2 Vectors

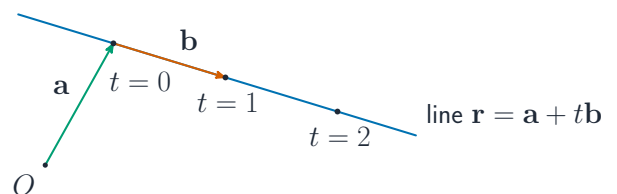
Pure Mathematics 3
Vectors

Vectors

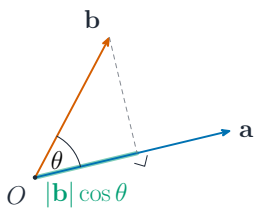
$|\mathbf{v}| = \sqrt{x^2 + y^2 + z^2}$; a **unit vector** 单位向量 has length 1.

$$\mathbf{r} = \mathbf{a} + t\mathbf{b} \text{ (a line).}$$

Lines can be parallel, intersect, or be **skew** 异面直线.



The scalar product



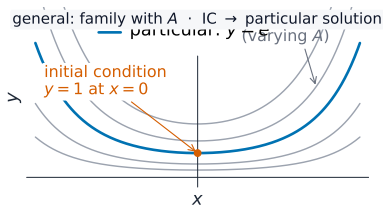
$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 = |\mathbf{a}| |\mathbf{b}| \cos \theta.$$

$$\mathbf{a} \cdot \mathbf{b} = 0 \Rightarrow \text{perpendicular.}$$

e.g. angle between $(1, 2, 2)$ and $(2, 2, 1)$: $\cos \theta = \frac{8}{9}$, $\theta = 27.3^\circ$.

3 Differential equations

Differential equations



Separable 可分离变量: split y and x , then integrate both sides.

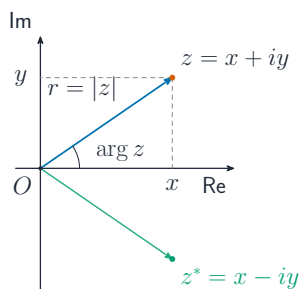
$$\frac{dy}{dx} = xy, \quad y = 1 \text{ at } x = 0$$

$$\ln y = \frac{1}{2}x^2 + c \Rightarrow y = e^{x^2/2}$$

A condition fixes C (the **particular solution** 特解).

4 Complex numbers

Complex numbers



$$z = x + iy, \quad i^2 = -1. \quad \text{Conjugate 共轭}$$

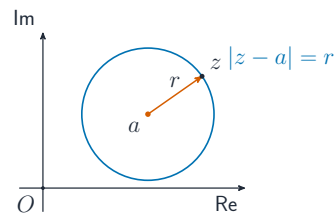
$$z^* = x - iy.$$

$$|z| = \sqrt{x^2 + y^2}, \quad z = re^{i\theta}.$$

Divide by multiplying by the conjugate:

$$\frac{3+i}{1-i} = 1+2i$$

Loci on the Argand diagram



A condition on z draws a **locus** 轨迹:

- $|z - a| = r$ - a circle
- $|z - a| = |z - b|$ - a perpendicular bisector
- $\arg(z - a) = \theta$ - a half-line

Worked example – integration by parts

Find $\int x \cos x \, dx$.

By parts with $u = x$, $\frac{dv}{dx} = \cos x$ (so $v = \sin x$):

$$\begin{aligned}\int x \cos x \, dx &= x \sin x - \int \sin x \, dx \\ &= x \sin x + \cos x + C\end{aligned}$$

Choose u to be the part that gets **simpler** when differentiated – here $x \rightarrow 1$. The other part must be integrable.

5

Recap

Topic 3 – key takeaways

1 Integration

partial fractions, by parts, substitution

2 Vectors

$\mathbf{r} = a + t\mathbf{b}$; $\mathbf{a} \cdot \mathbf{b} = 0$ perpendicular

3 Diff. equations

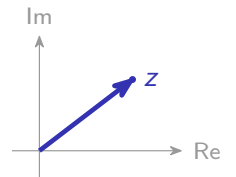
separate variables; a condition fixes C

4 Complex

$z = x + iy = re^{i\theta}$; loci on the Argand plane

Check yourself

- 1 What does $\mathbf{a} \cdot \mathbf{b} = 0$ tell you?
- 2 Which integration method suits $\int x \cos x \, dx$?
- 3 The conjugate of $3 - 2i$?
- 4 What locus is $|z - a| = r$?



Answers 1. they are perpendicular 2. by parts 3. $3 + 2i$ 4. a circle, centre a , radius r