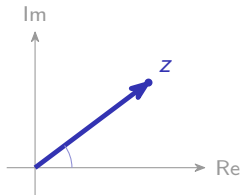


Pure Mathematics 3

A-Level Mathematics ■ Topic 3

A-Level Mathematics



What we will cover

- 1 Partial fractions & integration
- 2 Vectors
- 3 Differential equations
- 4 Complex numbers
- 5 Recap

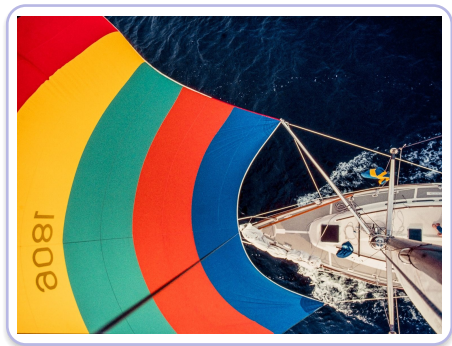


1

Algebra and integration

Partial fractions and integration

$$\frac{x+4}{(x+1)(x-2)} \xrightarrow{\text{split}} \frac{2}{x-2} - \frac{1}{x+1}$$



Vectors sailing

2

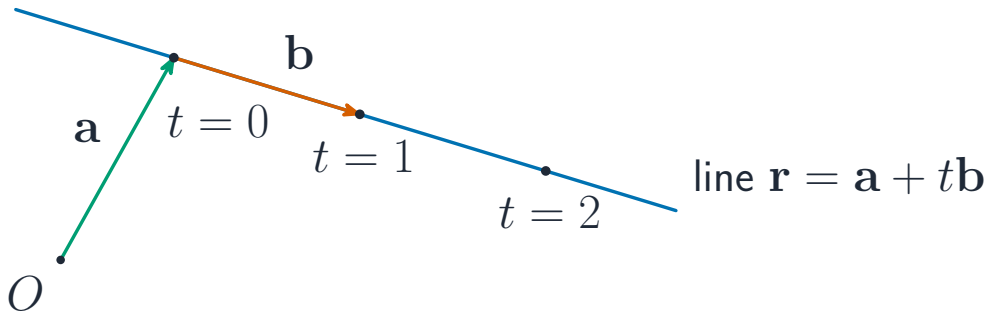
Vectors

Vectors

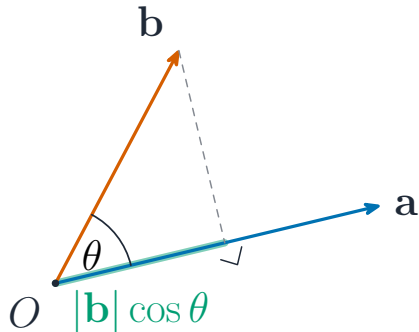
$|\mathbf{v}| = \sqrt{x^2 + y^2 + z^2}$; a **unit vector** 单位向量 has length 1.

$$\mathbf{r} = \mathbf{a} + t\mathbf{b} \text{ (a line).}$$

Lines can be parallel, intersect, or be **skew** 异面直线.



The scalar product



$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 = |\mathbf{a}| |\mathbf{b}| \cos \theta.$$

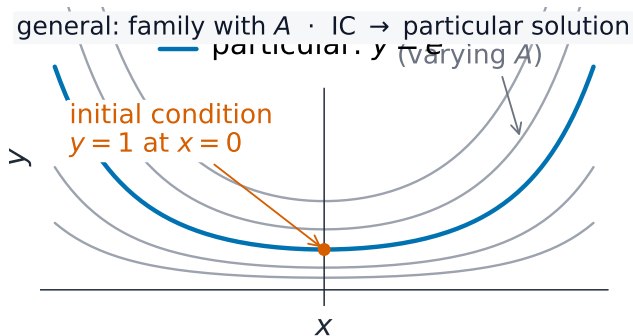
$$\mathbf{a} \cdot \mathbf{b} = 0 \Rightarrow \text{perpendicular.}$$

e.g. angle between $(1, 2, 2)$ and $(2, 2, 1)$: $\cos \theta = \frac{8}{9}$, $\theta = 27.3^\circ$.

3

Differential equations

Differential equations



Separable 可分离变量: split y and x , then integrate both sides.

$$\frac{dy}{dx} = xy, \quad y = 1 \text{ at } x = 0$$

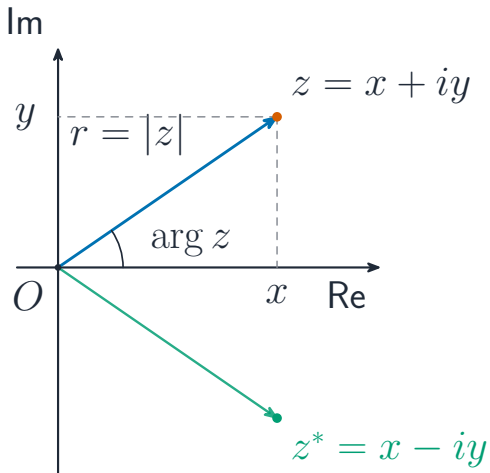
$$\ln y = \frac{1}{2}x^2 + c \Rightarrow y = e^{x^2/2}$$

A condition fixes C (the **particular solution** 特解).

4

Complex numbers

Complex numbers



$z = x + iy$, $i^2 = -1$. **Conjugate** 共轭
 $z^* = x - iy$.

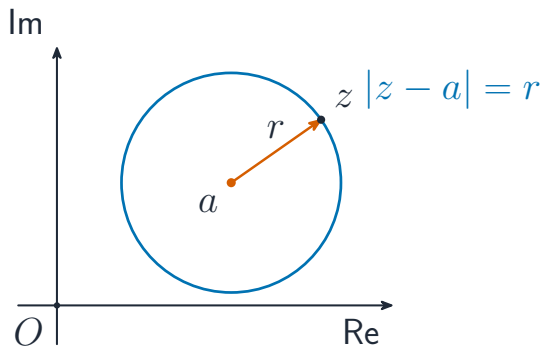
$$|z| = \sqrt{x^2 + y^2}, \quad z = re^{i\theta}.$$

Divide by multiplying by the conjugate:

$$\frac{3 + i}{1 - i}$$

$$= 1 + 2i$$

Loci on the Argand diagram



A condition on z draws a **locus** 轨迹:

- $|z - a| = r$ – a circle
- $|z - a| = |z - b|$ – a perpendicular bisector
- $\arg(z - a) = \theta$ – a half-line

Worked example – integration by parts

Find $\int x \cos x \, dx$.

By parts with $u = x$, $\frac{dv}{dx} = \cos x$ (so $v = \sin x$):

$$\begin{aligned}\int x \cos x \, dx &= x \sin x - \int \sin x \, dx \\ &= x \sin x + \cos x + C\end{aligned}$$

Choose u to be the part that gets **simpler** when differentiated – here $x \rightarrow 1$. The other part must be integrable.

5

Recap

Topic 3 – key takeaways

1

Integration

partial fractions, by parts,
substitution

2

Vectors

$\mathbf{r} = \mathbf{a} + t\mathbf{b}$; $\mathbf{a} \cdot \mathbf{b} = 0$ perpendicular

3

Diff. equations

separate variables; a condition
fixes C

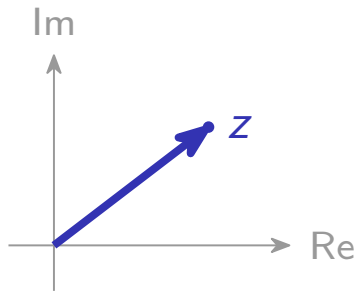
4

Complex

$z = x + iy = re^{i\theta}$; loci on the
Argand plane

Check yourself

- 1 What does $\mathbf{a} \cdot \mathbf{b} = 0$ tell you?
- 2 Which integration method suits $\int x \cos x \, dx$?
- 3 The conjugate of $3 - 2i$?
- 4 What locus is $|z - a| = r$?



Answers 1. they are perpendicular 2. by parts 3. $3 + 2i$ 4. a circle, centre a , radius r