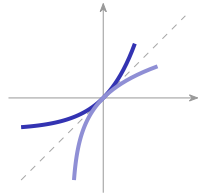


Pure Mathematics 2

A-Level Mathematics ■ Topic 2

A-Level Mathematics



Pure Mathematics 2

What we will cover

- 1 Modulus & polynomials
- 2 Logarithms & exponentials
- 3 Trigonometry
- 4 Differentiation
- 5 Integration & numerical methods
- 6 Recap



1 Algebra

Modulus and polynomials

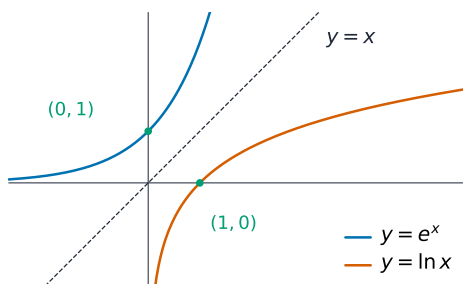
$|x|$ is x if $x \geq 0$ and $-x$ if $x < 0$. The graph of $y = |f(x)|$ reflects anything below the x -axis above it. To solve $|f(x)| = k$, do $f(x) = k$ and $f(x) = -k$. Sketch first – a

V-shape, vertex on the axis.

2

Logs and exponentials

Logarithms and exponentials



Laws of logs 对数定律:

$$\log mn = \log m + \log n,$$

$$\log m^k = k \log m.$$

e^x and $\ln x$ are inverses. Unknown in the power $\Rightarrow$ take logs.

Linear form 线性形式:

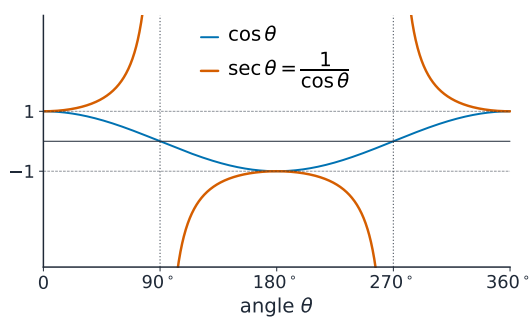
$$y = Ax^n \Rightarrow \ln y = \ln A + n \ln x.$$

3 Trigonometry

Pure Mathematics 2

Trigonometry

More trigonometry



Reciprocals: $\sec = \frac{1}{\cos}$,
 $\csc = \frac{1}{\sin}$, $\cot = \frac{1}{\tan}$.

$$\sec^2 \theta \equiv 1 + \tan^2 \theta.$$

Compound & **double**
angle 二倍角;

R-formula 辅助角公式

$$a \sin \theta + b \cos \theta = R \sin(\theta + \alpha).$$

4 Calculus

Differentiation

Standard: $\frac{d}{dx}e^x = e^x$, $\frac{d}{dx}\ln x = \frac{1}{x}$,
 $\frac{d}{dx}\sin x = \cos x$.

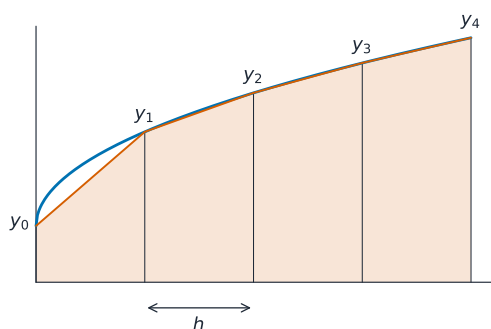
Two rules

product 乘积法则: $(uv)' = u'v + uv'$

quotient 商法则: $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$

Also **parametric** 参数方程 & **implicit** 隐式.

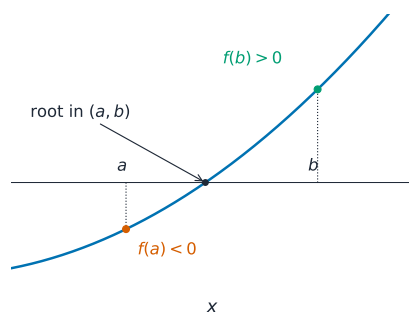
Integration



Reverse each derivative, e.g.
 $\int e^{ax+b} dx = \frac{1}{a}e^{ax+b} + C$.

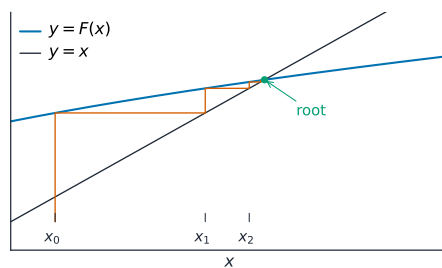
Trapezium rule 梯形法则
 estimates an integral: $\frac{h}{2}[y_0 + y_n + 2(y_1 + \dots + y_{n-1})]$.

Numerical solution of equations



sign change 变号 traps a root

If $f(a)$, $f(b)$ have opposite signs a root lies between; iterate until it **converges** 收敛.



iteration 迭代 $x_{n+1} = F(x_n)$

Worked example – the factor theorem

$p(x) = 2x^4 + kx^3 + kx^2 + 17x + 18$ has factor $(x + 2)$. Find k .

Factor theorem: $p(-2) = 0$.

$$2(16) - 8k + 4k - 34 + 18 = 16 - 4k = 0$$

$$\Rightarrow k = 4$$

$(x + 2)$ a factor $\Leftrightarrow p(-2) = 0$
– substitute the value that makes the bracket zero, not +2.

5

Recap

Topic 2 – key takeaways

1 Algebra

factor theorem; modulus inequalities

2 Logs

e^x & $\ln x$ inverse; take logs for powers

3 Trig

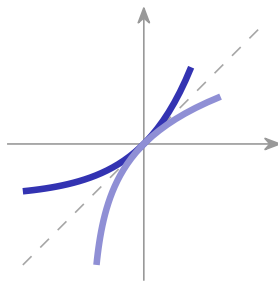
$\sec^2 = 1 + \tan^2$; R-formula

4 Calculus

product/quotient rules; trapezium rule

Check yourself

- 1 What is the remainder when $p(x) \div (x - a)$?
- 2 The derivative of $\ln x$?
- 3 Rewrite $\sec^2 \theta$ using $\tan$.
- 4 What does a sign change between a and b show?



Answers 1. $p(a)$ 2. $1/x$ 3. $1 + \tan^2 \theta$ 4. a root lies between them