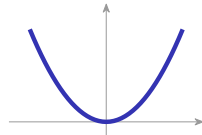


Pure Mathematics 1

A-Level Mathematics ■ Topic 1

A-Level Mathematics



Pure Mathematics 1

What we will cover

- 1 Quadratics & functions
- 2 Coordinate geometry
- 3 Circular measure & trigonometry
- 4 Series
- 5 Differentiation & integration
- 6 Recap



1

Quadratics and functions

Quadratics

$ax^2 + bx + c = 0$ has discriminant $\Delta = b^2 - 4ac$.

$$\Delta > 0$$

two distinct real roots

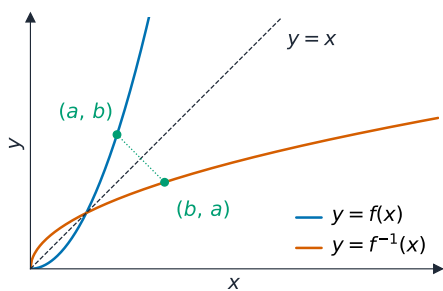
$$\Delta = 0$$

one repeated real root

$$\Delta < 0$$

no real roots

Functions



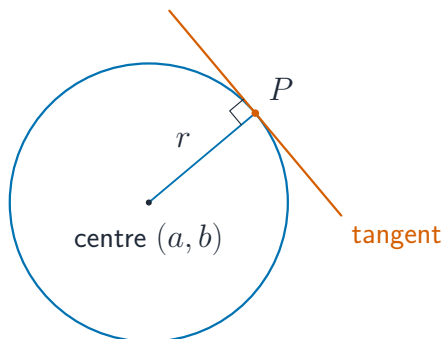
f^{-1} reflects f in $y = x$

- **domain** 定义域 (inputs), **range** 值域 (outputs)
- **one-one** 一一对应 $\Rightarrow$ has an **inverse** 反函数
- **composition** 复合函数
 $fg(x) = f(g(x))$
- **transformations** 变换: $f(x) + a$, $f(x + a)$, $af(x)$, $f(ax)$

2

Coordinate geometry

Coordinate geometry



Line gradient $m = \frac{y_2 - y_1}{x_2 - x_1}$;
perpendicular $\Rightarrow m_1 m_2 = -1$.

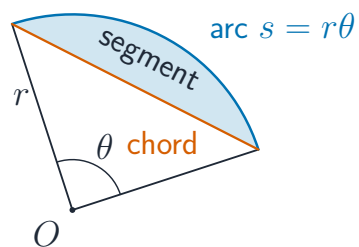
$$(x - a)^2 + (y - b)^2 = r^2.$$

A **tangent** 切线 meets the **radius** 半径 at a right angle.

3

Circular measure and trigonometry

Circular measure

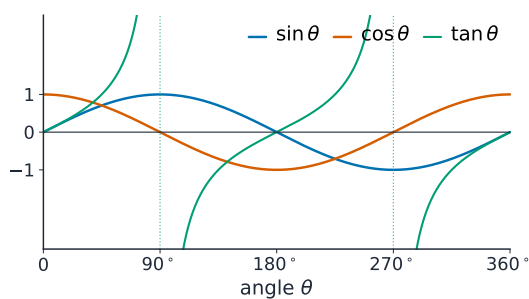


Radians 弧度: $\pi = 180^\circ$. For a **sector** 扇形 (angle in radians):

$$s = r\theta, \quad A = \frac{1}{2}r^2\theta.$$

Segment 弓形 $= \frac{1}{2}r^2(\theta - \sin \theta)$.

Trigonometry



$\sin, \cos$ repeat every 360° ; learn exact values.

Identities 恒等式: $\tan \theta \equiv \frac{\sin \theta}{\cos \theta}$, $\sin^2 \theta + \cos^2 \theta \equiv 1$.

Reduce to one function, then solve in the interval.

4 Series

Series

Binomial 二项展开式:

$$(a + b)^n = \sum \binom{n}{r} a^{n-r} b^r.$$

AP 等差数列: $u_n = a + (n - 1)d$.

GP 等比数列: $u_n = ar^{n-1}$.

Sum to infinity ($|r| < 1$)

$$S_\infty = \frac{a}{1 - r}.$$

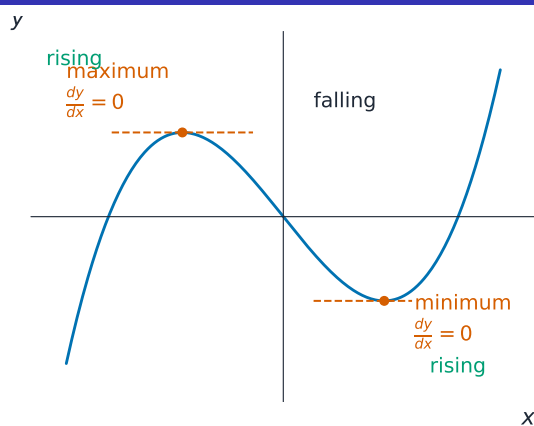
e.g. $a = 32$, $r = -\frac{3}{4}$: $S_\infty = \frac{128}{7}$.

5 Calculus

Pure Mathematics 1

└ Calculus

Differentiation



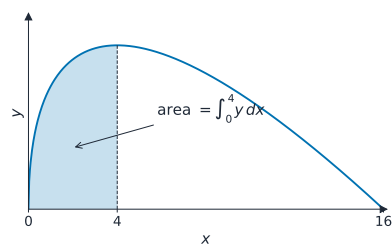
$$\frac{d}{dx}x^n = nx^{n-1}; \quad \text{chain: } f(g)g'.$$

Stationary point 驻点:
 $\frac{dy}{dx} = 0$. Then $f'' > 0$ min,
 $f'' < 0$ max.

Pure Mathematics 1

└ Calculus

Integration

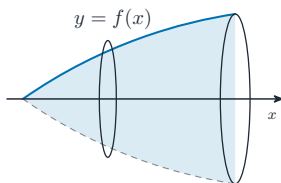


Reverse of **differentiation** 微分
(+C):

$$\int (ax + b)^n dx = \frac{(ax + b)^{n+1}}{a(n+1)} + C.$$

$$\text{Area} = \int_p^q y dx.$$

Volume of revolution



rotate the region around the x -axis to sweep a solid

Rotate a **region** 区域 about the x -axis – each slice is a disc of area πy^2 :

$$V = \pi \int_p^q y^2 dx.$$

Worked example – completing the square

Write $9x^2 - 36x + 8$ as $p(x + q)^2 + r$.

Take out 9 from the first two terms, complete the square inside:

$$\begin{aligned} 9(x^2 - 4x) + 8 &= 9((x - 2)^2 - 4) + 8 \\ &= 9(x - 2)^2 - 28 \end{aligned}$$

Factor out the x^2 coefficient **before** completing the square. The form gives the minimum instantly: $(2, -28)$.

Worked example – a maximum by differentiation

$y = 4x^{1/2} - x$ has a maximum at $x = a$. Find a .

$$\begin{aligned} \frac{dy}{dx} &= 2x^{-1/2} - 1 = 0 \Rightarrow x^{1/2} = 2 \\ \text{so } a &= 4 \text{ (and } y = 4 \cdot 2 - 4 = 4) \end{aligned}$$

Rewrite roots as **powers** ($\sqrt{x} = x^{1/2}$) before differentiating. Set $dy/dx = 0$ for a turning point.

6

Recap

Pure Mathematics 1

└ Recap

Topic 1 – key takeaways

1 Quadratics

discriminant decides the roots

2 Geometry

$m_1 m_2 = -1$; tangent $\perp$ radius

3 Trig

$\sin^2 + \cos^2 = 1$; sectors in radians

4 Calculus

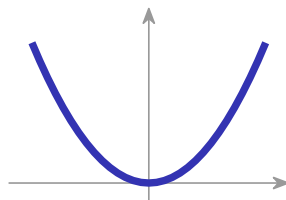
$\frac{dy}{dx} = 0$ stationary; $\int y dx$ area

Pure Mathematics 1

└ Recap

Check yourself

- 1 How many roots if $b^2 - 4ac < 0$?
- 2 Reflect a graph in which line to get its inverse?
- 3 Arc length of a sector (radians)?
- 4 Test for a minimum stationary point?



Answers 1. none 2. $y = x$ 3. $s = r\theta$ 4. $f'(x) > 0$