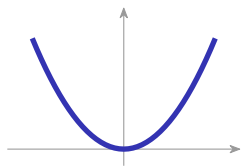
# Pure Mathematics 1

A-Level Mathematics ▪ Topic 1

A-Level Mathematics

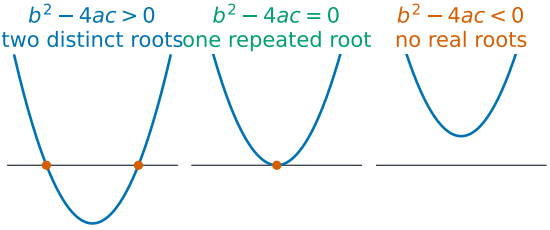


# What we will cover

1. Quadratics & functions
2. Coordinate geometry
3. Circular measure & trigonometry
4. Series
5. Differentiation & integration
6. Recap

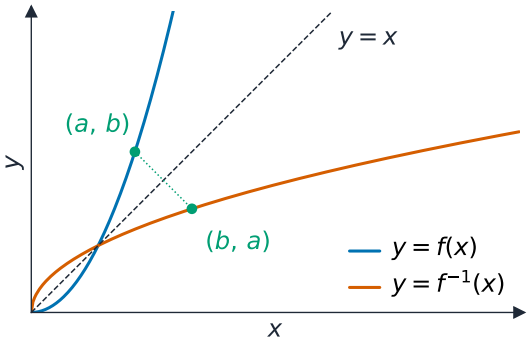
# 1 Quadratics and functions

# Quadratics




*Trig ferris*

# Functions

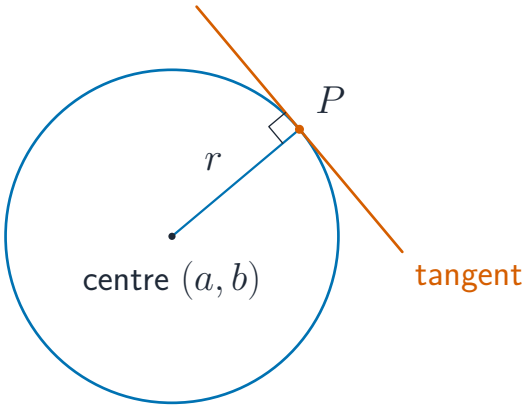



$f^{-1}$ *reflects* $f$ *in* $y = x$

- **domain** 定义域 (inputs), **range** 值域 (outputs)
- **one-one** 一一对应 $\Rightarrow$ has an **inverse** 反函数
- **composition** 复合函数 $fg(x) = f(g(x))$
- **transformations** 变换: $f(x) + a$, $f(x + a)$, $af(x)$, $f(ax)$

# 2 Coordinate geometry

# Coordinate geometry



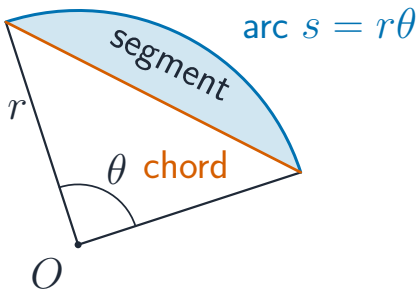

Line gradient $m = \dfrac{y_2 - y_1}{x_2 - x_1}$;
perpendicular $\Rightarrow m_1 m_2 = -1$.

$$(x - a)^2 + (y - b)^2 = r^2.$$

A **tangent** 切线 meets the **radius** 半径 at a right angle.

# 3 Circular measure and trigonometry

# Circular measure

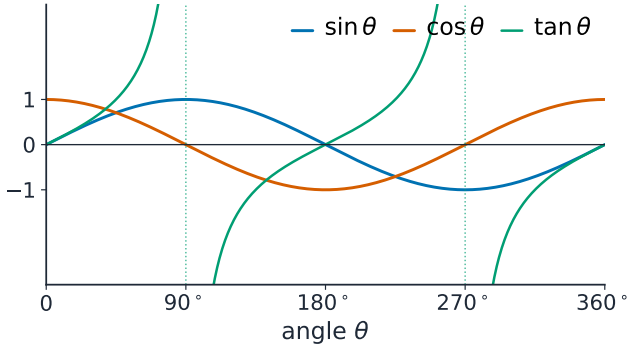



**Radians** 弧度: $\pi = 180°$. For a **sector** 扇形 (angle in radians):

$$s = r\theta, \quad A = \frac{1}{2}r^2\theta.$$

**Segment** 弓形 $= \frac{1}{2}r^2(\theta - \sin\theta)$.

# Trigonometry




$\sin, \cos$ repeat every $360°$; learn exact values.

**Identities** 恒等式: $\tan\theta \equiv \dfrac{\sin\theta}{\cos\theta}$, $\sin^2\theta + \cos^2\theta \equiv 1$.

Reduce to one function, then solve in the interval.

# 4 Series

# Series

**Binomial** 二项展开式:

$$(a+b)^n = \sum \binom{n}{r} a^{n-r} b^r.$$

**AP** 等差数列: $u_n = a + (n-1)d$.
**GP** 等比数列: $u_n = ar^{n-1}$.

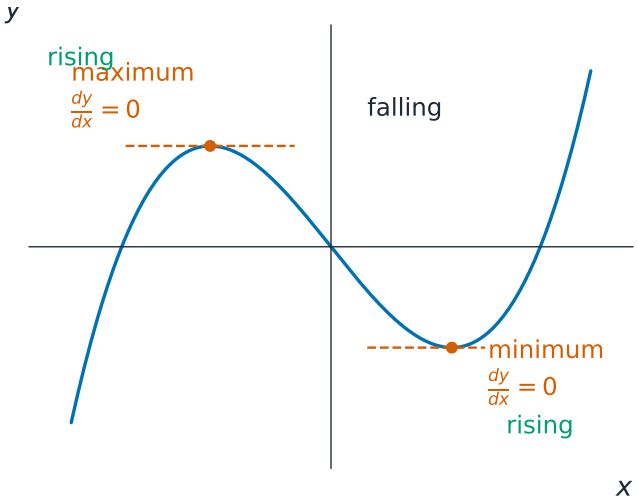
**Sum to infinity ($|r| < 1$)**

$$S_\infty = \frac{a}{1-r}.$$

e.g. $a = 32$, $r = -\frac{3}{4}$: $S_\infty = \frac{128}{7}$.

# 5 Calculus

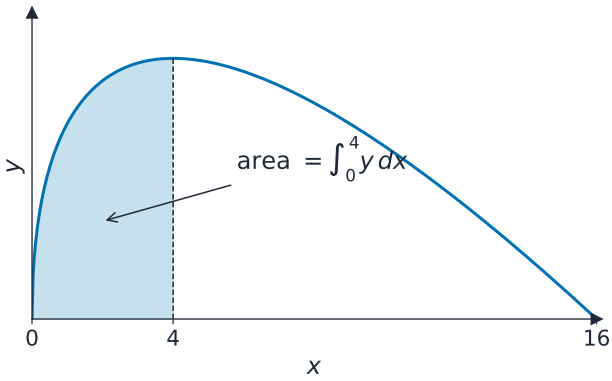
# Differentiation




$$\frac{d}{dx}x^n = nx^{n-1}; \quad \text{chain: } f'(g)\,g'.$$

**Stationary point** 驻点:
$\frac{dy}{dx} = 0$. Then $f'' > 0$ min, $f'' < 0$ max.

# Integration

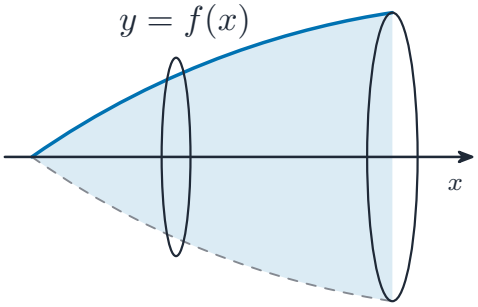



Reverse of differentiation $(+C)$:

$$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C.$$

$$\text{Area} = \int_p^q y\,dx.$$

# Volume of revolution




rotate the region around the $x$-axis to sweep a solid

Rotate a region about the $x$-axis – each slice is a disc of area $\pi y^2$:

$$V = \pi \int_p^q y^2 \, dx.$$

# Worked example – completing the square

**Write $9x^2 - 36x + 8$ as $p(x+q)^2 + r$.**

Take out $9$ from the first two terms, complete the square inside:

$$9(x^2 - 4x) + 8 = 9\big((x-2)^2 - 4\big) + 8$$
$$= 9(x-2)^2 - 28$$

Factor out the $x^2$ coefficient before completing the square. The form gives the minimum instantly: $(2, -28)$.

# Worked example – a maximum by differentiation

**$y = 4x^{1/2} - x$ has a maximum at $x = a$. Find $a$.**

$$\frac{dy}{dx} = 2x^{-1/2} - 1 = 0 \Rightarrow x^{1/2} = 2$$

so $a = 4$ (and $y = 4 \cdot 2 - 4 = 4$)

Rewrite roots as powers ($\sqrt{x} = x^{1/2}$) before differentiating. Set $dy/dx = 0$ for a turning point.

# 6 Recap

# Topic 1 – key takeaways

**1 Quadratics**

discriminant decides the roots

**2 Geometry**

$m_1 m_2 = -1$; tangent $\perp$ radius

**3 Trig**

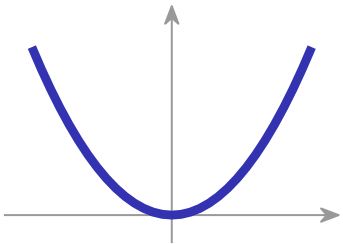
$\sin^2 + \cos^2 = 1$; sectors in radians

**4 Calculus**

$\frac{dy}{dx} = 0$ stationary; $\int y\,dx$ area

# Check yourself

1. How many roots if $b^2 - 4ac < 0$?
2. Reflect a graph in which line to get its inverse?
3. Arc length of a sector (radians)?
4. Test for a minimum stationary point?



**Answers** **1.** none **2.** $y = x$ **3.** $s = r\theta$ **4.** $f'(x) > 0$