Official mark schemes compress a solution into "M1 A1" and leave the reasoning to you. These are the same problems written out the way you should write them in the exam: every step visible, the method marks earned before the answer, and a note on where the marks actually sit. 官方答案只给 M1 A1, 这里把每一步都写出来。

Pure 1 – quadratics, functions and coordinate geometry

Express $2x^2-12x+23$ in the form $a(x+b)^2+c$, and hence state the least value of the expression and the value of x at which it occurs. [5]

$2x^2-12x+23 = 2(x^2-6x)+23 = 2\left[(x-3)^2-9\right]+23 = 2(x-3)^2-18+23 = 2(x-3)^2+5$. Since $(x-3)^2 \geq 0$ for all real x, the least value is 5, occurring at $x=3$.

Take the 2 out of the FIRST TWO terms only. Factoring it out of the constant as well is the commonest error here and loses every subsequent mark.

The method marks are for the completed square inside the bracket and for expanding back out correctly. Writing the final form with no working scores the answer mark only.

'Hence' means use the form you just found. Differentiating instead is a valid method but the question did not ask for it, and in a 'hence' question it may not be credited.

The function f is defined by $f(x)=x^2-4x+7$ for $x \geq 2$. Find $f^{-1}(x)$ and state its domain. [5]

Complete the square: $f(x)=(x-2)^2+3$. Let $y=(x-2)^2+3$, so $(x-2)^2=y-3$ and $x-2=\sqrt{y-3}$, taking the POSITIVE root because $x \geq 2$. Hence $x = 2+\sqrt{y-3}$, so $f^{-1}(x)=2+\sqrt{x-3}$. The range of f is $f(x) \geq 3$, so the domain of f^{-1} is $x \geq 3$.

The sign choice is a mark: $\sqrt{\ }$ has two roots and the given domain $x \geq 2$ is what selects the positive one. An answer with $\pm$ is not a function.

The domain of the inverse is the RANGE of the original – find the range first, from the completed square, and the domain follows.

The line $y=2x+k$ is a tangent to the curve $y=x^2-3x+7$. Find k. [4]

Substituting: $x^2-3x+7=2x+k$, so $x^2-5x+(7-k)=0$. A tangent touches at exactly one point, so the discriminant is zero: $(-5)^2-4(1)(7-k)=0$, giving $25-28+4k=0$, so $4k=3$ and $k=\frac{3}{4}$.

The mark is for KNOWING that tangency means $b^2-4ac=0$ and saying so. Setting up the quadratic without stating why the discriminant vanishes leaves a mark on the table.

Watch the sign of c: it is $7-k$, not $7+k$.

Pure 1 – trigonometry and series

Solve $3\sin^2\theta = 4\cos\theta$ for $0° \leq \theta \leq 360°$. [5]

Use $\sin^2\theta = 1-\cos^2\theta$: $3(1-\cos^2\theta)=4\cos\theta$, so $3\cos^2\theta+4\cos\theta-3=0$. This is a quadratic in $\cos\theta$; by the formula $\cos\theta = \dfrac{-4\pm\sqrt{16+36}}{6} = \dfrac{-4\pm\sqrt{52}}{6}$, giving $\cos\theta = 0.5352$ or $\cos\theta=-1.868$. The second is impossible since $|\cos\theta| \leq 1$. From $\cos\theta = 0.5352$, $\theta = 57.6°$ or $\theta = 302.4°$.

Converting to a single trigonometric function is the first method mark; leaving both sin and cos in the equation makes it unsolvable.

REJECTING the impossible root explicitly is a mark. Silently ignoring it is not the same as rejecting it.

The second solution comes from the symmetry $360° - 57.6°$. Giving only the principal value loses the final mark on almost every trigonometric equation.

The first three terms of a geometric progression are $k+4$, k, and $2k-9$. Find the possible values of k. [4]

In a geometric progression the ratio of consecutive terms is constant, so $\dfrac{k}{k+4} = \dfrac{2k-9}{k}$. Cross-multiplying: $k^2=(k+4)(2k-9) = 2k^2-9k+8k-36 = 2k^2-k-36$. Hence $0 = k^2-k-36$... rearranged, $k^2-k-36=0$, so $k = \dfrac{1\pm\sqrt{1+144}}{2} = \dfrac{1\pm\sqrt{145}}{2}$, giving $k=6.52$ or $k=-5.52$.

Setting the two ratios equal is the whole method – one mark for the statement, one for forming the quadratic, one for solving, one for both values.

Expand carefully: $(k+4)(2k-9)$ has a $-k$ term, not $+k$. A sign slip here still earns the method marks if the working is visible.

Pure 1 – differentiation and integration

A curve has equation $y=\dfrac{8}{x}+2x$. Find the coordinates of the stationary point and determine its nature. [6]

Write $y = 8x^{-1}+2x$, so $\dfrac{dy}{dx} = -8x^{-2}+2$. At a stationary point $\dfrac{dy}{dx}=0$: $2=\dfrac{8}{x^2}$, so $x^2=4$ and $x=\pm 2$. When $x=2$, $y=4+4=8$; when $x=-2$, $y=-4-4=-8$. Now $\dfrac{d^2y}{dx^2} = 16x^{-3}$. At $x=2$ this is $+2>0$, a MINIMUM at $(2,8)$; at $x=-2$ it is $-2<0$, a MAXIMUM at $(-2,-8)$.

Rewrite as a power of x before differentiating – the quotient rule is not needed and using it here wastes time and invites error.

Both roots of $x^2=4$ matter. Giving only $x=2$ loses half the marks in a question that asks for the stationary point(s).

The nature mark requires a REASON: the sign of the second derivative, stated. Writing 'minimum' with no justification earns nothing.

Find the area of the region enclosed by the curve $y=4x-x^2$ and the x-axis. [5]

The curve meets the x-axis where $4x-x^2=0$, that is $x(4-x)=0$, so $x=0$ and $x=4$. The area is $\displaystyle\int_0^4 (4x-x^2)\,dx = \left[2x^2-\tfrac{1}{3}x^3\right]_0^4 = \left(32-\tfrac{64}{3}\right)-0 = \tfrac{96-64}{3} = \tfrac{32}{3}$.

Finding the limits from the roots is the first mark; integrating with the limits guessed earns nothing even if the integral is right.

Show the substituted bracket before evaluating. That line is the accuracy mark, and it survives an arithmetic slip in the final subtraction.

Statistics 1

The random variable X has a normal distribution with mean 74 and standard deviation 12. Find $P(X>90)$. [3]

Standardise: $z=\dfrac{90-74}{12} = \dfrac{16}{12} = 1.333$. Then $P(X>90) = P(Z>1.333) = 1-\Phi(1.333) = 1-0.9088 = 0.0912$.

The standardisation line is the method mark and must be visible, with the correct mean and standard deviation the right way round.

Φ tables give the area to the LEFT, so a 'greater than' probability needs the $1-$. Forgetting it is the most common error in the whole topic.

Quote z to three significant figures before reading the table; rounding to 1.3 changes the third decimal place of the answer.

A committee of 5 is chosen from 6 men and 8 women. Find the number of committees containing at least 3 women. [4]

Count the cases. Exactly 3 women: $\binom{8}{3}\binom{6}{2} = 56 \times 15 = 840$. Exactly 4 women: $\binom{8}{4}\binom{6}{1} = 70 \times 6 = 420$. Exactly 5 women: $\binom{8}{5}\binom{6}{0} = 56 \times 1 = 56$. Total $= 840 + 420 + 56 = 1316$.

'At least 3' means three separate cases, added. One mark is for identifying the cases, and it is lost by answering only the first.

Each case multiplies a women-choice by a men-choice – the two selections are independent, so they multiply; the cases are exclusive, so they add. Getting these two operations the wrong way round is the classic error.