A-Level Mathematics

Revision Sheet · 考前速记

Learn these by heart before the exam. 考试前把这些内容背熟。

Algebra and functions

b^2-4ac >0 two real roots, $=0$ one repeated root, <0 none 判别式

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ 求根公式

Factor and remainder theorem 因式与余数定理: if $f(a)=0$ then $(x-a)$ is a factor; dividing $f(x)$ by $(x-a)$ leaves remainder $f(a)$

$|x|<a \iff -a<x<a$ 绝对值

$\log_a x+\log_a y=\log_a(xy)$ 对数

$\log_a x^k=k\log_a x$, $a^x=e^{x\ln a}$ solve $a^x=b$ by taking logs of both sides

Sequences and binomial

$u_n=a+(n-1)d$, $S_n=\frac{n}{2}(2a+(n-1)d)$ 等差数列

$u_n=ar^{n-1}$, $S_n=\dfrac{a(1-r^n)}{1-r}$ 等比数列

$S_\infty=\dfrac{a}{1-r}$, $|r|<1$ sum to infinity, converges only when $|r|<1$

$(a+b)^n=\sum_{r=0}^{n}\binom{n}{r}a^{n-r}b^r$ 二项式定理

$(1+x)^n=1+nx+\frac{n(n-1)}{2!}x^2+\cdots$ n any rational; valid for $|x|<1$

Trigonometry

$\sin^2\theta+\cos^2\theta=1$ 三角恒等式

$1+\tan^2\theta=\sec^2\theta$, $1+\cot^2\theta=\csc^2\theta$

$\sin 2\theta=2\sin\theta\cos\theta$

$\cos 2\theta=2\cos^2\theta-1=1-2\sin^2\theta$

$a\sin\theta+b\cos\theta=R\sin(\theta+\alpha)$ $R=\sqrt{a^2+b^2}$, $\tan\alpha=b/a$

arc $s=r\theta$, area $=\frac{1}{2}r^2\theta$ θ in radians 弧度制

Differentiation and integration

$\dfrac{d}{dx}(x^n)=nx^{n-1}$ 导数

$\dfrac{d}{dx}(e^{kx})=ke^{kx}$, $\dfrac{d}{dx}(\ln x)=\frac{1}{x}$

$\dfrac{d}{dx}(\sin x)=\cos x$, $\dfrac{d}{dx}(\cos x)=-\sin x$, $\dfrac{d}{dx}(\tan x)=\sec^2 x$

product: $(uv)'=u'v+uv'$ 乘积法则

quotient: $\left(\frac{u}{v}\right)'=\dfrac{u'v-uv'}{v^2}$ 商法则

chain: $\dfrac{dy}{dx}=\dfrac{dy}{du}\cdot\dfrac{du}{dx}$ 链式法则

Stationary point where $\frac{dy}{dx}=0$; $\frac{d^2y}{dx^2}>0$ *minimum,* <0 *maximum.*

$\int x^n\,dx=\dfrac{x^{n+1}}{n+1}+c$, $n\neq-1$ 积分

$\int\frac{1}{x}\,dx=\ln|x|+c$, $\int e^{kx}\,dx=\frac{1}{k}e^{kx}+c$

Area $=\int_a^b y\,dx$, $V=\pi\int_a^b y^2\,dx$ area under a curve; volume of revolution about the x-axis

Trapezium rule 梯形法则:
$\int_a^b y\,dx\approx\frac{h}{2}[y_0+2(y_1+\cdots+y_{n-1})+y_n]$, $h=\frac{b-a}{n}$

Vectors and numerical methods

$|\mathbf{a}|=\sqrt{x^2+y^2+z^2}$ 向量的模

$\mathbf{a}\cdot\mathbf{b}=|\mathbf{a}||\mathbf{b}|\cos\theta$ $=0$ when the vectors are perpendicular 点积

$\mathbf{r}=\mathbf{a}+t\mathbf{d}$ vector equation of a line through $\mathbf{a}$, direction $\mathbf{d}$

Newton-Raphson 牛顿法: $x_{n+1}=x_n-\dfrac{f(x_n)}{f'(x_n)}$ – iterate to a root

A sign change of $f(x)$ *over* $[a,b]$ *(continuous) locates a root between them.*

Mechanics

$v=u+at$, $s=ut+\frac{1}{2}at^2$, $v^2=u^2+2as$ constant acceleration (suvat) only 匀加速公式

$F=ma$ resultant force; resolve forces along and perpendicular to motion

$F\le\mu R$ friction; $F=\mu R$ at the point of slipping 摩擦力

$P=Fv$ power = force × velocity (driving force) 功率

$W=Fd\cos\theta$, $E_k=\frac{1}{2}mv^2$, $E_p=mgh$
work-energy: work done = change in kinetic energy

Projectile: horizontal velocity constant; vertical acceleration $=-g$ *(9.8* m/s^2*).*

Statistics

$\bar{x}=\dfrac{\sum x}{n}$, $\sigma^2=\dfrac{\sum x^2}{n}-\bar{x}^2$ 均值与方差

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$ 概率加法

$P(A\cap B)=P(A)\,P(B\mid A)$ independent when $P(A\cap B)=P(A)P(B)$

$X\sim B(n,p)$: $P(X=r)=\binom{n}{r}p^r(1-p)^{n-r}$ mean np, variance $np(1-p)$ 二项分布

$X\sim N(\mu,\sigma^2)$: $Z=\dfrac{X-\mu}{\sigma}$ standardise, then read $\Phi(z)$ from tables 正态分布

Normal approximates $B(n,p)$ *when* n *large,* $np>5$ *and* $n(1-p)>5$ *(use a continuity correction).*