A-Level Mathematics

Learn these by heart before the exam. 考试前把这些内容背熟。

Algebra and functions

$b^2 - 4ac$ > 0 two real roots, $= 0$ one repeated root, < 0 none 判别式

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ 求根公式

Factor and remainder theorem 因式与余数定理: if $f(a) = 0$ then $(x-a)$ is a factor; dividing $f(x)$ by $(x-a)$ leaves remainder $f(a)$

$|x| < a \iff -a < x < a$ 绝对值

$\log_a x + \log_a y = \log_a(xy)$ 对数

$\log_a x^k = k \log_a x, \quad a^x = e^{x \ln a}$ solve $a^x = b$ by taking logs of both sides

Sequences and binomial

$u_n = a + (n-1)d, \quad S_n = \frac{n}{2}(2a + (n-1)d)$ 等差数列

$$u_n = ar^{n-1}, \quad S_n = \frac{a(1-r^n)}{1-r}$$ 等比数列

$$S_\infty = \frac{a}{1-r}, \quad |r| < 1$$ sum to infinity, converges only when $|r| < 1$

$(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r$ 二项式定理

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \cdots$ n any rational; valid for $|x| < 1$

Trigonometry

$\sin^2\theta + \cos^2\theta = 1$ 三角恒等式

$1 + \tan^2\theta = \sec^2\theta, \quad 1 + \cot^2\theta = \csc^2\theta$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

$a\sin\theta + b\cos\theta = R\sin(\theta + \alpha)$ $R = \sqrt{a^2+b^2}$, $\tan\alpha = b/a$

arc $s = r\theta, \quad$ area $= \frac{1}{2}r^2\theta$ θ in radians 弧度制

Differentiation and integration

$$\frac{d}{dx}(x^n) = nx^{n-1}$$ 导数

$$\frac{d}{dx}(e^{kx}) = ke^{kx}, \quad \frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}(\sin x) = \cos x, \quad \frac{d}{dx}(\cos x) = -\sin x, \quad \frac{d}{dx}(\tan x) = \sec^2 x$$

product: $(uv)' = u'v + uv'$ 乘积法则

quotient: $$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$ 商法则

chain: $$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$ 链式法则

Stationary point where $\frac{dy}{dx} = 0$*;* $\frac{d^2y}{dx^2} > 0$ *minimum,* < 0 *maximum.*

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c, \quad n \neq -1$$ 积分

$\int \frac{1}{x} \, dx = \ln|x| + c, \quad \int e^{kx} \, dx = \frac{1}{k}e^{kx} + c$

Area $= \int_a^b y \, dx, \quad V = \pi \int_a^b y^2 \, dx$ area under a curve; volume of revolution about the x-axis

Trapezium rule 梯形法则: $\int_a^b y \, dx \approx \frac{h}{2}[y_0 + 2(y_1 + \cdots + y_{n-1}) + y_n]$, $h = \frac{b-a}{n}$

Vectors and numerical methods

$|\mathbf{a}| = \sqrt{x^2 + y^2 + z^2}$ 向量的模

$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$ $= 0$ when the vectors are perpendicular 点积

$\mathbf{r} = \mathbf{a} + t\mathbf{d}$ vector equation of a line through $\mathbf{a}$, direction $\mathbf{d}$

Newton-Raphson 牛顿法: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$ – iterate to a root

A sign change of $f(x)$ *over* $[a, b]$ *(continuous) locates a root between them.*

Mechanics

$v = u + at, \quad s = ut + \frac{1}{2}at^2, \quad v^2 = u^2 + 2as$ constant acceleration (suvat) only 匀加速公式

$F = ma$ resultant force; resolve forces along and perpendicular to motion

$F \leq \mu R$ friction; $F = \mu R$ at the point of slipping 摩擦力

$P = Fv$ power = force × velocity (driving force) 功率

$W = Fd\cos\theta, \quad E_k = \frac{1}{2}mv^2, \quad E_p = mgh$ work-energy: work done = change in kinetic energy

Projectile: horizontal velocity constant; vertical acceleration $= -g$ *(9.8 m/s*^{2}*).*

Statistics

$$\bar{x} = \frac{\sum x}{n}, \quad \sigma^2 = \frac{\sum x^2}{n} - \bar{x}^2$$ 均值与方差

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ 概率加法

$P(A \cap B) = P(A)\,P(B \mid A)$ independent when $P(A \cap B) = P(A)P(B)$

$X \sim B(n, p): \; P(X = r) = \binom{n}{r}p^r(1-p)^{n-r}$ mean np, variance $np(1-p)$ 二项分布

$$X \sim N(\mu, \sigma^2): \; Z = \frac{X - \mu}{\sigma}$$ standardise, then read $\Phi(z)$ from tables 正态分布

Normal approximates $B(n,p)$ *when* n *large,* $np > 5$ *and* $n(1-p) > 5$ *(use a continuity correction).*