A-Level Mathematics

Syllabus Checklist · 大纲自查表

Tick one box per statement: ☐ not yet ☐ nearly ☐ confident. 每条打一个勾：尚未掌握 / 快会了 / 有把握。

1. Pure Mathematics 1 纯数学 1

1.1 Quadratics 二次函数

- ☐☐☐ carry out the process of **completing the square** for a quadratic polynomial $ax^2 + bx + c$ and use a completed square form
- ☐☐☐ find the **discriminant** of a quadratic polynomial $ax^2 + bx + c$ and use the discriminant
- ☐☐☐ solve **quadratic equations**, and **quadratic inequalities**, in one unknown
- ☐☐☐ solve by substitution a pair of **simultaneous equations** of which one is linear and one is quadratic
- ☐☐☐ recognise and solve equations in x which are quadratic in some function of x.

1.2 Functions 函数

- ☐☐☐ understand the terms **function**, **domain**, **range**, **one-one function**, **inverse function** and **composition of functions**
- ☐☐☐ identify the **range** of a given function in simple cases, and find the **composition** of two given functions
- ☐☐☐ determine whether or not a given function is **one-one**, and find the **inverse** of a one-one function in simple cases
- ☐☐☐ illustrate in graphical terms the relation between a **one-one function** and its **inverse**
- ☐☐☐ understand and use the **transformations** of the graph of $y = f(x)$ given by $y = f(x) + a$, $y = f(x + a)$, $y = af(x)$, $y = f(ax)$ and simple combinations of these.

1.3 Coordinate geometry 坐标几何

- ☐☐☐ find the equation of a **straight line** given sufficient information
- ☐☐☐ interpret and use any of the forms $y = mx + c$, $y - y_1 = m(x - x_1)$, $ax + by + c = 0$ in solving problems
- ☐☐☐ understand that the equation $(x - a)^2 + (y - b)^2 = r^2$ represents the **circle** with centre (a, b) and radius r
- ☐☐☐ use algebraic methods to solve problems involving lines and circles
- ☐☐☐ understand the relationship between a graph and its associated algebraic equation, and use the relationship between points of intersection of graphs and solutions of equations.

1.4 Circular measure 弧度制

- ☐☐☐ understand the definition of a **radian**, and use the relationship between radians and degrees
- ☐☐☐ use the formulae $s = r\theta$ and $A = \frac{1}{2}r^2\theta$ in solving problems concerning the **arc length** and **sector area** of a circle.

1.5 Trigonometry 三角学

- ☐☐☐ sketch and use graphs of the **sine**, **cosine** and **tangent** functions (for angles of any size, and using either degrees or radians)
- ☐☐☐ use the exact values of the **sine**, **cosine** and **tangent** of 30°, 45°, 60°, and related angles

☐☐☐ use the notations $\sin^{-1} x$, $\cos^{-1} x$, $\tan^{-1} x$ to denote the **principal values** of the **inverse trigonometric relations**

☐☐☐ use the identities $\frac{\sin\theta}{\cos\theta} \equiv \tan\theta$ and $\sin^2\theta + \cos^2\theta \equiv 1$

☐☐☐ find all the solutions of simple **trigonometrical equations** lying in a specified interval (general forms of solution are not included).

1.6 Series 级数

☐☐☐ use the expansion of $(a+b)^n$, where n is a positive integer

☐☐☐ recognise **arithmetic** and **geometric progressions**

☐☐☐ use the formulae for the nth term and for the sum of the first n terms to solve problems involving **arithmetic** or **geometric progressions**

☐☐☐ use the condition for the **convergence** of a **geometric progression**, and the formula for the **sum to infinity** of a **convergent geometric progression**.

1.7 Differentiation 微分

☐☐☐ understand the **gradient** of a curve at a point as the limit of the gradients of a suitable sequence of chords, and use the notations $f'(x)$, $f''(x)$, $\frac{\mathrm{d}y}{\mathrm{d}x}$, and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ for first and second **derivatives**

☐☐☐ use the **derivative** of x^n (for any rational n), together with constant multiples, sums and differences of functions, and of composite functions using the **chain rule**

☐☐☐ apply differentiation to **gradients**, **tangents** and **normals**, increasing and decreasing functions and **rates of change**

☐☐☐ locate **stationary points** and determine their nature, and use information about stationary points in sketching graphs.

1.8 Integration 积分

☐☐☐ understand **integration** as the reverse process of differentiation, and integrate $(ax+b)^n$ (for any rational n except -1), together with constant multiples, sums and differences

☐☐☐ solve problems involving the evaluation of a **constant of integration**

☐☐☐ evaluate **definite integrals**

☐☐☐ use definite integration to find: - the **area** of a region bounded by a curve and lines parallel to the axes, or between a curve and a line or between two curves - a **volume of revolution** about one of the axes.

2. Pure Mathematics 2 纯数学 2

2.1 Algebra 代数

☐☐☐ understand the meaning of \

☐☐☐ divide a **polynomial**, of degree not exceeding 4, by a linear or quadratic **polynomial**, and identify the **quotient** and **remainder** (which may be zero)

☐☐☐ use the **factor theorem** and the **remainder theorem**.

2.2 Logarithmic and exponential functions 对数函数与指数函数

☐☐☐ understand the relationship between **logarithms** and **indices**, and use the laws of **logarithms** (excluding change of base)

☐☐☐ understand the definition and properties of e^x and $\ln x$, including their relationship as **inverse functions** and their graphs

☐☐☐ use **logarithms** to solve equations and inequalities in which the unknown appears in **indices**

☐☐☐ use **logarithms** to transform a given relationship to linear form, and hence determine unknown constants by considering the gradient and/or intercept.

2.3 Trigonometry 三角学

☐☐☐ understand the relationship of the **secant**, **cosecant** and **cotangent** functions to cosine, sine and tangent, and use properties and graphs of all six trigonometric functions for angles of any magnitude

☐☐☐ use trigonometrical identities for the simplification and exact evaluation of expressions, and in the course of solving equations, and select an identity or identities appropriate to the context, showing familiarity in particular with the use of: – $\sec^2\theta \equiv 1 + \tan^2\theta$ and $\csc^2\theta \equiv 1 + \cot^2\theta$ – the expansions of $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$ – the formulae for $\sin 2A$, $\cos 2A$ and $\tan 2A$ – the expression of $a\sin\theta + b\cos\theta$ in the forms $R\sin(\theta \pm \alpha)$ and $R\cos(\theta \pm \alpha)$.

2.4 Differentiation 微分

☐☐☐ use the derivatives of e^x, $\ln x$, $\sin x$, $\cos x$, $\tan x$, together with constant multiples, sums, differences and composites

☐☐☐ differentiate products and quotients

☐☐☐ find and use the first derivative of a function which is defined parametrically or implicitly.

2.5 Integration 积分

☐☐☐ • extend the idea of 'reverse differentiation' to include the integration of e^{ax+b}, $\frac{1}{ax+b}$, $\sin(ax+b)$, $\cos(ax+b)$ and $\sec^2(ax+b)$

☐☐☐ • use trigonometrical relationships in carrying out integration

☐☐☐ • understand and use the **trapezium rule** to estimate the value of a definite integral.

2.6 Numerical solution of equations 方程的数值解

☐☐☐ • locate approximately a root of an equation, by means of graphical considerations and/or searching for a **sign change**

☐☐☐ • understand the idea of, and use the notation for, a sequence of approximations which converges to a root of an equation

☐☐☐ • understand how a given simple **iterative formula** of the form $x_{n+1} = \mathrm{F}(x_n)$ relates to the equation being solved, and use a given iteration, or an iteration based on a given rearrangement of an equation, to determine a root to a prescribed degree of accuracy.

3. Pure Mathematics 3 纯数学 3

3.1 Algebra 代数

☐☐☐ • understand the meaning of \

☐☐☐ • divide a **polynomial**, of degree not exceeding 4, by a linear or quadratic polynomial, and identify the **quotient** and **remainder** (which may be zero)

☐☐☐ • use the **factor theorem** and the **remainder theorem**

☐☐☐ • recall an appropriate form for expressing rational functions in **partial fractions**, and carry out the decomposition, in cases where the denominator is no more complicated than – $(ax+b)(cx+d)(ex+f)$ – $(ax+b)(cx+d)^2$ – $(ax+b)(cx^2+d)$

☐☐☐ • use the expansion of $(1+x)^n$, where n is a rational number and \

3.2 Logarithmic and exponential functions 对数函数与指数函数

☐☐☐ understand the relationship between **logarithms** and **indices**, and use the laws of logarithms (excluding change of base)

☐☐☐ understand the definition and properties of e^x and $\ln x$, including their relationship as **inverse functions** and their graphs

☐☐☐ use logarithms to solve equations and inequalities in which the unknown appears in indices

☐☐☐ use logarithms to transform a given relationship to linear form, and hence determine unknown constants by considering the **gradient** and/or **intercept**.

3.3 Trigonometry 三角学

☐☐☐ understand the relationship of the **secant**, **cosecant** and **cotangent** functions to cosine, sine and tangent, and use properties and graphs of all six trigonometric functions for angles of any magnitude

☐☐☐ use trigonometrical identities for the simplification and exact evaluation of expressions, and in the course of solving equations, and select an identity or identities appropriate to the context, showing familiarity in particular with the use of: – $\sec^2\theta \equiv 1 + \tan^2\theta$ and ${}^2\theta \equiv 1 + \cot^2\theta$ – the expansions of $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$ – the formulae for $\sin 2A$, $\cos 2A$ and $\tan 2A$ – the expression of $a\sin\theta + b\cos\theta$ in the forms $R\sin(\theta \pm \alpha)$ and $R\cos(\theta \pm \alpha)$.

3.4 Differentiation 微分

☐☐☐ use the derivatives of e^x, $\ln x$, $\sin x$, $\cos x$, $\tan x$, $\tan^{-1} x$, together with constant multiples, sums, differences and composites

☐☐☐ differentiate **products** and **quotients**

☐☐☐ find and use the first derivative of a function which is defined **parametrically** or **implicitly**.

3.5 Integration 积分

☐☐☐ extend the idea of 'reverse differentiation' to include the integration of e^{ax+b}, $\frac{1}{ax+b}$, $\sin(ax+b)$, $\cos(ax+b)$, $\sec^2(ax+b)$ and $\frac{1}{x^2+a^2}$

☐☐☐ use trigonometrical relationships in carrying out integration

☐☐☐ integrate rational functions by means of decomposition into **partial fractions**

☐☐☐ recognise an integrand of the form $\frac{kf'(x)}{f(x)}$, and integrate such functions

☐☐☐ recognise when an integrand can usefully be regarded as a product, and use **integration by parts**

☐☐☐ use a given **substitution** to simplify and evaluate either a definite or an indefinite integral.

3.6 Numerical solution of equations 方程的数值解

☐☐☐ • locate approximately a **root** of an equation, by means of graphical considerations and/or searching for a **sign change**

☐☐☐ • understand the idea of, and use the notation for, a sequence of approximations which **converges** to a root of an equation

☐☐☐ • understand how a given simple **iterative formula** of the form $x_{n+1} = \mathrm{F}(x_n)$ relates to the equation being solved, and use a given **iteration**, or an iteration based on a given rearrangement of an equation, to determine a root to a prescribed degree of accuracy.

3.7 Vectors 向量

☐☐☐ • use standard notations for **vectors**, i.e. $\begin{pmatrix} x \\ y \end{pmatrix}$, $x\mathbf{i} + y\mathbf{j}$, $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$, $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, $\overrightarrow{AB}$, $\mathbf{a}$

☐☐☐ • carry out addition and subtraction of vectors and multiplication of a vector by a **scalar**, and interpret these operations in geometrical terms

☐☐☐ • calculate the **magnitude** of a vector, and use **unit vectors**, **displacement vectors** and **position vectors**

☐☐☐ • understand the significance of all the symbols used when the equation of a straight line is expressed in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$, and find the equation of a line, given sufficient information

☐☐☐ • determine whether two lines are **parallel**, intersect or are **skew**, and find the point of intersection of two lines when it exists

☐☐☐ • use formulae to calculate the **scalar product** of two vectors, and use scalar products in problems involving lines and points.

3.8 Differential equations 微分方程

☐☐☐ formulate a simple statement involving a **rate of change** as a **differential equation**

☐☐☐ find by integration a general form of solution for a **first order differential equation** in which the variables are **separable**

☐☐☐ use an **initial condition** to find a **particular solution**

☐☐☐ interpret the solution of a **differential equation** in the context of a problem being modelled by the equation.

3.9 Complex numbers 复数

☐☐☐ understand the idea of a **complex number**, recall the meaning of the terms **real part**, **imaginary part**, **modulus**, **argument**, **conjugate**, and use the fact that two complex numbers are equal if and only if both real and imaginary parts are equal

☐☐☐ carry out operations of addition, subtraction, multiplication and division of two complex numbers expressed in **Cartesian form** $x + \mathrm{i}y$

☐☐☐ use the result that, for a polynomial equation with real coefficients, any non-real roots occur in **conjugate pairs**

☐☐☐ represent complex numbers geometrically by means of an **Argand diagram**

☐☐☐ carry out operations of multiplication and division of two complex numbers expressed in **polar form** $r(\cos\theta + \mathrm{i}\sin\theta) \equiv r\mathrm{e}^{\mathrm{i}\theta}$

☐☐☐ find the two **square roots** of a complex number

☐☐☐ understand in simple terms the geometrical effects of conjugating a complex number and of adding, subtracting, multiplying and dividing two complex numbers

☐☐☐ illustrate simple equations and inequalities involving complex numbers by means of **loci** in an **Argand diagram**

4. Mechanics 力学

4.1 Forces and equilibrium 力与平衡

☐☐☐ identify the forces acting in a given situation

☐☐☐ understand the vector nature of force, and find and use components and resultants

☐☐☐ use the principle that, when a particle is in equilibrium, the vector sum of the forces acting is zero, or equivalently, that the sum of the components in any direction is zero

☐☐☐ understand that a contact force between two surfaces can be represented by two components, the normal component and the frictional component

☐☐☐ use the model of a 'smooth' contact, and understand the limitations of this model

☐☐☐ understand the concepts of **limiting friction** and **limiting equilibrium**, recall the definition of

coefficient of friction, and use the relationship $F = \mu R$ or $F \leqslant \mu R$, as appropriate

☐☐☐ use Newton's third law.

4.2 Kinematics of motion in a straight line 直线运动的运动学

☐☐☐ understand the concepts of **distance** and **speed** as **scalar** quantities, and of **displacement**, **velocity** and **acceleration** as **vector** quantities

☐☐☐ sketch and interpret **displacement–time graphs** and **velocity–time graphs**, and in particular appreciate that – the area under a velocity–time graph represents displacement, – the gradient of a displacement–time graph represents velocity, – the gradient of a velocity–time graph represents acceleration

☐☐☐ use **differentiation** and **integration** with respect to time to solve simple problems concerning displacement, velocity and acceleration

☐☐☐ use appropriate formulae for motion with **constant acceleration** in a straight line.

4.3 Momentum 动量

☐☐☐ use the definition of **linear momentum** and show understanding of its vector nature

☐☐☐ use **conservation of linear momentum** to solve problems that may be modelled as the direct impact of two bodies.

4.4 Newton's laws of motion 牛顿运动定律

☐☐☐ • apply **Newton's laws of motion** to the linear motion of a particle of constant mass moving under the action of constant forces, which may include **friction**, **tension** in an inextensible string and **thrust** in a connecting rod

☐☐☐ • use the relationship between **mass** and **weight**

☐☐☐ • solve simple problems which may be modelled as the motion of a particle moving vertically or on an inclined plane with constant **acceleration**

☐☐☐ • solve simple problems which may be modelled as the motion of **connected particles**.

4.5 Energy, work and power 能量、功与功率

☐☐☐ • understand the concept of the **work done** by a force, and calculate the work done by a constant force when its point of application undergoes a displacement not necessarily parallel to the force

☐☐☐ • understand the concepts of **gravitational potential energy** and **kinetic energy**, and use appropriate formulae

☐☐☐ • understand and use the relationship between the change in energy of a system and the work done by the external forces, and use in appropriate cases the principle of **conservation of energy**

☐☐☐ • use the definition of **power** as the rate at which a force does work, and use the relationship between power, force and velocity for a force acting in the direction of motion

☐☐☐ • solve problems involving, for example, the **instantaneous acceleration** of a car moving on a hill against a resistance.

5. Probability & Statistics 1 概率与统计 1

5.1 Representation of data 数据的表示

☐☐☐ • select a suitable way of presenting raw statistical data, and discuss advantages and/or disadvantages that particular representations may have

☐☐☐ • draw and interpret **stem-and-leaf diagrams**, **box-and-whisker plots**, **histograms** and **cumulative frequency graphs**

☐☐☐ • understand and use different measures of **central tendency** (mean, median, mode) and **variation** (range, interquartile range, standard deviation)

☐☐☐ • use a cumulative frequency graph

☐☐☐ • calculate and use the **mean** and **standard deviation** of a set of data (including grouped data) either from the data itself or from given totals Σx and Σx^2, or coded totals $\Sigma(x-a)$ and $\Sigma(x-a)^2$, and use such totals in solving problems which may involve up to two data sets.

5.2 Permutations and combinations 排列与组合

☐☐☐ • understand the terms **permutation** and **combination**, and solve simple problems involving selections

☐☐☐ • solve problems about arrangements of objects in a line, including those involving – repetition (e.g. the number of ways of arranging the letters of the word 'NEEDLESS') – restriction (e.g. the number of ways several people can stand in a line if two particular people must, or must not, stand next to each other).

5.3 Probability 概率

☐☐☐ evaluate probabilities in simple cases by means of enumeration of equiprobable elementary events, or by calculation using **permutations** or **combinations**

☐☐☐ use addition and multiplication of probabilities, as appropriate, in simple cases

☐☐☐ understand the meaning of **exclusive** and **independent** events, including determination of whether events A and B are independent by comparing the values of $\mathrm{P}(A \cap B)$ and $\mathrm{P}(A) \times \mathrm{P}(B)$

☐☐☐ calculate and use **conditional probabilities** in simple cases.

5.4 Discrete random variables 离散随机变量

☐☐☐ draw up a **probability distribution** table relating to a given situation involving a **discrete random variable** X, and calculate $\mathrm{E}(X)$ and $\mathrm{Var}(X)$

☐☐☐ use formulae for probabilities for the **binomial** and **geometric** distributions, and recognise practical situations where these distributions are suitable models

☐☐☐ use formulae for the expectation and variance of the binomial distribution and for the expectation of the geometric distribution.

5.5 The normal distribution 正态分布

☐☐☐ understand the use of a **normal distribution** to model a **continuous random variable**, and use normal distribution tables

☐☐☐ solve problems concerning a variable X, where $X \sim N(\mu, \sigma^2)$, including: – finding the value of $P(X > x_1)$, or a related probability, given the values of x_1, μ, σ. – finding a relationship between x_1, μ and σ given the value of $P(X > x_1)$ or a related probability

☐☐☐ recall conditions under which the **normal distribution** can be used as an **approximation** to the **binomial distribution**, and use this approximation, with a **continuity correction**, in solving problems.

6. Probability & Statistics 2 概率与统计 2

6.1 The Poisson distribution 泊松分布

☐☐☐ • use formulae to calculate probabilities for the distribution $\mathrm{Po}(\lambda)$

☐☐☐ • use the fact that if $X \sim \mathrm{Po}(\lambda)$ then the mean and variance of X are each equal to λ

☐☐☐ • understand the relevance of the **Poisson distribution** to the distribution of random events, and use the **Poisson distribution** as a model

☐☐☐ • use the **Poisson distribution** as an approximation to the **binomial distribution** where appropriate

☐☐☐ • use the **normal distribution**, with **continuity correction**, as an approximation to the **Poisson distribution** where appropriate.

6.2 Linear combinations of random variables 随机变量的线性组合

☐☐☐ • use, when solving problems, the results that – $\mathrm{E}(aX+b) = a\mathrm{E}(X)+b$ and $\mathrm{Var}(aX+b) = a^2\mathrm{Var}(X)$ – $\mathrm{E}(aX+bY) = a\mathrm{E}(X)+b\mathrm{E}(Y)$ – $\mathrm{Var}(aX+bY) = a^2\mathrm{Var}(X)+b^2\mathrm{Var}(Y)$ for independent X and Y – if X has a **normal distribution** then so does $aX+b$ – if X and Y have independent **normal distributions** then $aX+bY$ has a **normal distribution** – if X and Y have independent **Poisson distributions** then $X+Y$ has a **Poisson distribution**.

6.3 Continuous random variables 连续随机变量

☐☐☐ • understand the concept of a **continuous random variable**, and recall and use properties of a **probability density function**

☐☐☐ • use a **probability density function** to solve problems involving probabilities, and to calculate the **mean** and **variance** of a distribution.

6.4 Sampling and estimation 抽样与估计

☐☐☐ • understand the distinction between a **sample** and a **population**, and appreciate the necessity for **randomness** in choosing samples

☐☐☐ • explain in simple terms why a given **sampling method** may be unsatisfactory

☐☐☐ • recognise that a **sample mean** can be regarded as a **random variable**, and use the facts that $\mathrm{E}(\overline{X}) = \mu$ and that $\mathrm{Var}(\overline{X}) = \frac{\sigma^2}{n}$

☐☐☐ • use the fact that $\overline{X}$ has a **normal distribution** if X has a normal distribution

☐☐☐ • use the **Central Limit Theorem** where appropriate

☐☐☐ • calculate **unbiased estimates** of the population **mean** and **variance** from a sample, using either raw or summarised data

☐☐☐ • determine and interpret a **confidence interval** for a population mean in cases where the population is normally distributed with known variance or where a large sample is used

☐☐☐ • determine, from a large sample, an approximate **confidence interval** for a **population proportion**.

6.5 Hypothesis tests 假设检验

☐☐☐ • understand the nature of a **hypothesis test**, the difference between **one-tailed** and **two-tailed** tests, and the terms **null hypothesis**, **alternative hypothesis**, **significance level**, **rejection region** (or **critical region**), **acceptance region** and **test statistic**

☐☐☐ • formulate hypotheses and carry out a **hypothesis test** in the context of a single observation from a population which has a **binomial** or **Poisson distribution**, using – direct evaluation of probabilities – a normal approximation to the binomial or the Poisson distribution, where appropriate

☐☐☐ • formulate hypotheses and carry out a **hypothesis test** concerning the population mean in cases where the population is normally distributed with known variance or where a large sample is used

☐☐☐ • understand the terms **Type I error** and **Type II error** in relation to hypothesis tests

☐☐☐ • calculate the probabilities of making **Type I** and **Type II errors** in specific situations involving tests based on a normal distribution or direct evaluation of binomial or Poisson probabilities.