

1(a)

$$4k + k + 4k + 9k = 1, k = \frac{1}{18}$$

x	-2	1	2	3
$P(X = x)$	$\frac{4}{18}$	$\frac{1}{18}$	$\frac{4}{18}$	$\frac{9}{18}$

1(b)

$$[E(X) = 28k =] \frac{14}{9}, 1.56$$

1(c)

$$P(X \neq 2 | X > 0) = \frac{\frac{10}{18}}{\frac{14}{18}}$$

$$= \frac{5}{7}, 0.714$$

5(a)

Method 1

$$[P(5, 6, 7) =] {}^7C_5 (0.6)^5(0.4)^2 + {}^7C_6 (0.6)^6(0.4)^1 + (0.6)^7$$

$$[= 0.261274 + 0.130637 + 0.027994]$$

$$= 0.420$$

Method 2

$$[1 - P(0,1,2,3,4) =] 1 - \{(0.4)^7 + {}^7C_1 (0.6) (0.4)^6 + {}^7C_2 (0.6)^2(0.4)^5$$

$$+ {}^7C_3 (0.6)^3(0.4)^4 + {}^7C_4 (0.6)^4(0.4)^3\}$$

$$= 0.420$$

5(b)

$$[\text{Mean} = 0.1 \times 150 =] 15$$

$$[\text{Variance} = 0.1 \times 150 \times 0.9 =] 13.5$$

$$[P(X < 22) = P(Z < \frac{21.5 - 15}{\sqrt{13.5}})]$$

$$[P(Z < 1.769) = \Phi(1.769)]$$

$$0.962$$

1(a)	$[(0.6)^4 (0.4) =] 0.05184, \frac{162}{3125}$
1(b)	$\left[1 - (1 - 0.6^6) =\right] 0.6^6$ Or $1 - (0.4 + 0.4 \times 0.6 + 0.4 \times 0.6^2 + 0.4 \times 0.6^3 + 0.4 \times 0.6^4 + 0.4 \times 0.6^5)$
	$= 0.0467, \frac{729}{15625}$

2(a)

x	1	2	3	4	5
$P(X=x)$	$\frac{5}{15}$	$\frac{4}{15}$	$\frac{3}{15}$	$\frac{2}{15}$	$\frac{1}{15}$
	$\frac{40}{120}$	$\frac{32}{120}$	$\frac{24}{120}$	$\frac{16}{120}$	$\frac{8}{120}$
	$\frac{1}{3}$		$\frac{1}{5}$		
	0.3333	0.2667	0.2[000]	0.1333	0.06667

All decimals correct to at least 3SF

2(b)

 $[E(X) =]$

$$1 \times \frac{5}{15} + 2 \times \frac{4}{15} + 3 \times \frac{3}{15} + 4 \times \frac{2}{15} + 5 \times \frac{1}{15}$$

$$\left[\frac{5+8+9+8+5}{15} = \frac{35}{15}, \frac{7}{3} \right]$$

 $[\text{Var}(X) =]$

$$1^2 \times \frac{5}{15} + 2^2 \times \frac{4}{15} + 3^2 \times \frac{3}{15} + 4^2 \times \frac{2}{15} + 5^2 \times \frac{1}{15}$$

$$- \left(\text{their } \frac{35}{15} \right)^2$$

$$\left[\frac{[1 \times] 5 + 4 \times 4 + 9 \times 3 + 16 \times 2 + 25 [\times 1]}{15} - \frac{49}{9} \right]$$

$$= \frac{14}{9}, 1.56$$

6(a)

$$\left[\left(\frac{5}{7} \right)^{10} = \right] 0.0346, \frac{9765625}{282475249}$$

6(b)

Method 1

$$[P(0, 1, 2) =] {}^{10}C_2 \left(\frac{6}{7}\right)^8 \left(\frac{1}{7}\right)^2 + {}^{10}C_1 \left(\frac{6}{7}\right)^9 \left(\frac{1}{7}\right) + \left(\frac{6}{7}\right)^{10}$$

$$[= 0.2675729 + 0.3567639 + 0.2140583 =]$$

0.838

$$[P(0,1,2) =] 1 - \left\{ \left(\frac{1}{7}\right)^{10} + {}^{10}C_9 \left(\frac{6}{7}\right) \left(\frac{1}{7}\right)^9 + {}^{10}C_1 \left(\frac{6}{7}\right)^2 \left(\frac{1}{7}\right)^8 + {}^{10}C_1$$

$$\left(\frac{6}{7}\right)^3 \left(\frac{1}{7}\right)^7 + {}^{10}C_1 \left(\frac{6}{7}\right)^4 \left(\frac{1}{7}\right)^6 + {}^{10}C_1 \left(\frac{6}{7}\right)^5 \left(\frac{1}{7}\right)^5 + {}^{10}C_1$$

$$\left(\frac{6}{7}\right)^4 \left(\frac{1}{7}\right)^6 + {}^{10}C_1 \left(\frac{6}{7}\right)^3 \left(\frac{1}{7}\right)^7 \right\}$$

0.838

6(c)

$$[\text{Mean} = 392 \times \frac{1}{7} =] 56$$

$$[\text{Variance} = 392 \times \frac{1}{7} \times \frac{6}{7} =] 48$$

$$[P(X > 65) = P(Z > \frac{65.5 - 56}{\sqrt{48}})]$$

$$[= 1 - \Phi(1.3712)]$$

$$= 1 - 0.9149$$

$$= 0.0851 \text{ final answer}$$

4(a)

$$P\left(\frac{-1.2}{2.5} < Z < \frac{1.2}{2.5}\right)$$

$$[= \Phi(0.48) + \Phi(-0.48) = 2\Phi(0.48) - 1]$$

$$2 \times 0.6844 - 1 \text{ or } 2 \times (0.6844 - 0.5) \text{ or } 0.6844 - 0.3156$$

$$0.3688$$

$$\text{Expected number} = 0.3688 \times 600 = 221.28 \text{ so } 221 \text{ or } 222$$

4(b)	Method 1
	$[P(2 \leq X < 8) = 1 - P(0, 1, 8, 9) =]$ $1 - (0.4^9 + {}^9C_1 0.4^8 0.6 + {}^9C_8 0.4^1 0.6^8 + 0.6^9) =$ $[1 - 0.000262144 - 0.00353894 - 0.060466176 - 0.010077696 =]$
	0.926
	Method 2
	$[P(2 \leq X < 8) = P(2, 3, 4, 5, 6, 7) =]$ ${}^9C_2 0.4^7 0.6^2 + {}^9C_3 0.4^6 0.6^3 + {}^9C_4 0.4^5 0.6^4 + {}^9C_5 0.4^4 0.6^5 + {}^9C_6 0.4^3 0.6^6 + {}^9C_7 0.4^2 0.6^7$
	0.926

4(c)	Mean = $80 \times 0.6 = 48$ Variance = $80 \times 0.6 \times 0.4 = 19.2$
	$[P(X > 50) =] P\left(Z > \frac{50.5 - 48}{\sqrt{19.2}}\right)$
	$[P(Z > 0.571) = 1 - \Phi(0.571) =]$ $1 - 0.7160 =$
	0.284

6(a)

Method 1

$$P(X=2) = \frac{6}{10} \times \frac{5}{9} \times \frac{4}{8} \times \frac{3}{7} \times \frac{4!}{2!2!} = \frac{3}{7}$$

Method 2

$$\frac{{}^6C_2 \times {}^4C_2}{{}^{10}C_4} = \left[\frac{15 \times 6}{210} \right] \frac{3}{7}$$

6(b)

x	0	1	2	3	4
$P(X=x)$	$\frac{1}{210},$ 0.00476	$\frac{4}{35},$ 0.114	$\frac{3}{7}$	$\frac{8}{21},$ 0.381	$\frac{1}{14},$ 0.0714

6(c)

$$[P(2B, 3B \mid 3B1R \text{ or } 2B2R \text{ or } 1B \ 3R) =] \frac{\frac{3}{7} + \frac{8}{21}}{\frac{4}{35} + \frac{3}{7} + \frac{8}{21}}$$

$$= \left[\frac{\frac{17}{21}}{\frac{97}{105}} \right] = \frac{170}{194}, \frac{85}{97}, 0.876$$