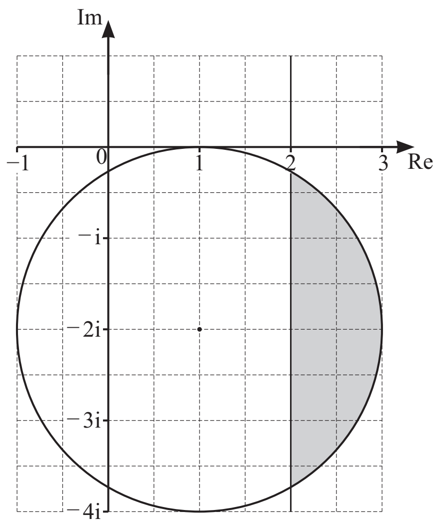
5



The shaded region on the Argand diagram shows points representing complex numbers z defined by two inequalities. The shaded region is bounded by a circle and a line parallel to the imaginary axis. The boundaries of the region are included in the shaded region.

(a) Find two inequalities in terms of z that define the shaded region. [3]

(b) Calculate the least value of $\arg z$ for points in this region. [3]

6 Solve the quadratic equation $(2+\mathrm{i})w^2+4w+2-\mathrm{i}=0$. Give your answers in the form $x+\mathrm{i}y$, where x and y are real. [5]

3 Find the complex numbers z for which $\frac{z+5\mathrm{i}}{z-5}$ is real and $|z|=\sqrt{17}$. Give your answers in the form $z=x+\mathrm{i}y$, where x and y are real. [6]

vel Mathematics: **9709 Mathematics June 2025 Question Paper 31**

6 It is given that $z_1 = 3e^{\frac{1}{4}\pi i}$, $z_2 = \frac{3}{2}e^{\frac{1}{6}\pi i}$ and $\omega = 2e^{\frac{1}{2}\pi i}$.

(a) State the values of ωz_1 and ωz_2. Give your answers in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$. [2]

....

....

....

....

....

(b) On a sketch of an Argand diagram with origin O, show the points A, B, C and D representing the complex numbers z_1, z_2, ωz_1 and ωz_2 respectively. [2]

(c) State the geometric effects of multiplying z_1 and z_2 by ω. [2]

....

....

....

....

....

....

....

3 On an Argand diagram shade the region whose points represent complex numbers z which satisfy both the inequalities $|z - 3i| \leqslant 2$ and $\frac{1}{4}\pi \leqslant \arg(z - 1 - 2i) \leqslant \frac{3}{4}\pi$. [5]

5 The square roots of $-1-4\sqrt{5}\,\mathrm{i}$ can be expressed in the Cartesian form $x+\mathrm{i}y$, where x and y are real and exact.

By first forming a quartic equation in x or y, find the square roots of $-1-4\sqrt{5}\,\mathrm{i}$ in exact Cartesian form. [5]