11 With respect to the origin O, the points A, B, C and D have position vectors given by

$$\overrightarrow{OA} = \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix}, \qquad \overrightarrow{OB} = \begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix}, \qquad \overrightarrow{OC} = \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix} \qquad \text{and} \qquad \overrightarrow{OD} = \begin{pmatrix} 3 \\ -5 \\ 4 \end{pmatrix}.$$

The line m passes through the points A and B.

(a) Find a vector equation for m. [2]

(b) Find the position vector of the point of intersection of m and the line passing through the points C and D. [4]

(c) Find the position vector of the foot of the perpendicular from C to m. [4]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

8 With respect to the origin O, the points A and B have position vectors $2\mathbf{i}+4\mathbf{k}$ and $5\mathbf{i}+\mathbf{j}+6\mathbf{k}$ respectively. The line l_1 passes through the points A and B.

(a) Find a vector equation for the line l_1. [2]

The line l_2 has equation $\mathbf{r} = 2\mathbf{i}+\mathbf{j}+5\mathbf{k}+\mu(\mathbf{i}+2\mathbf{j}+3\mathbf{k})$.

(b) Show that l_1 and l_2 do **not** intersect. [4]

(c) Find the acute angle between the directions of l_1 and l_2. [3]

9 With respect to the origin O, the points A, B and C have position vectors given by

$$\overrightarrow{OA} = \begin{pmatrix} 1 \\ -4 \\ 2 \end{pmatrix}, \qquad \overrightarrow{OB} = \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}.$$

(a) Find a vector equation for the line through A and B. [2]

(b) Using a scalar product, find the exact value of $\cos BAC$. [4]

(c) Hence find the exact area of triangle ABC. [3]