9 **(a)** By sketching a suitable pair of graphs, show that the equation

$$\sec 2x = -e^{x}$$

has only one root in the interval $0 < x < \frac{1}{2}\pi$. [2]

(b) Show by calculation that this root lies between 0.9 and 1. [2]

(c) Show that if a sequence of values given by the iterative formula

$$x_{n+1} = \tfrac{1}{2}\cos^{-1}\left(-e^{-x_n}\right)$$

converges, then it converges to the root of the equation in part **(a)**. [1]

(d) Use the iterative formula given in part **(c)** to calculate x correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

9 The constant a is such that $\int_1 6x \ln x \, dx = 4$.

(a) Show that $a = \exp\left(\frac{1}{6}\left(\frac{5}{a^2}+3\right)\right)$, where $\exp(x)$ denotes e^x. [5]

(b) Verify by calculation that a lies between 2 and 2.1. [2]

(c) Use an iterative formula based on the equation in part **(a)** to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

6 **(a)** By sketching a suitable pair of graphs, show that the equation

$$|x-2| = 2\sin\tfrac{1}{2}x$$

has only one root in the interval $0 < x < \pi$. [2]

(b) Show by calculation that this root lies between 1 and 1.5. [2]

(c) Use the iterative formula $x_{n+1} = 2 - 2\sin\tfrac{1}{2}x_n$ with an initial value of 1.03 to calculate the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]