

- 9 (a) By sketching a suitable pair of graphs, show that the equation

$$\sec 2x = -e^x$$

has only one root in the interval $0 < x < \frac{1}{2}\pi$.

[2]

- (b)** Show by calculation that this root lies between 0.9 and 1.

[2]

①

- (c) Show that if a sequence of values given by the iterative formula

$$x_{n+1} = \frac{1}{2} \cos^{-1}(-e^{-x_n})$$

converges, then it converges to the root of the equation in part (a).

[1]

- (d) Use the iterative formula given in part (c) to calculate x correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

[3]

②

9 The constant a is such that $\int_1^a 6x \ln x \, dx = 4$.

- (a) Show that $a = \exp\left(\frac{1}{6}\left(\frac{5}{a^2} + 3\right)\right)$, where $\exp(x)$ denotes e^x . [5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- (b) Verify by calculation that a lies between 2 and 2.1. [2]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

- (c) Use an iterative formula based on the equation in part (a) to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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- (c) Use the iterative formula $x_{n+1} = 2 - 2 \sin \frac{1}{2} x_n$ with an initial value of 1.03 to calculate the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

$$|x-2|=2\sin\frac{1}{2}x$$

has only one root in the interval $0 < x < \pi$.

[2]

- (b) Show by calculation that this root lies between 1 and 1.5.

[2]