7 The polynomial $\mathrm{p}(x)$ is defined by

$$\mathrm{p}(x) = 2x^4 + kx^3 + kx^2 + 17x + 18,$$

where k is a constant. It is given that $(x+2)$ is a factor of $\mathrm{p}(x)$.

(a) Find the value of k. [2]

It is given that the equation $\mathrm{p}(x) = 0$ has exactly two real roots, denoted by α and β, where α is an integer and β is not an integer.

(b) State the value of α and show that β satisfies the equation $x = \sqrt[3]{-2x - 4.5}$. [4]

(c) Show by calculation that $-1.4 < \beta < -1.0$. [2]

(d) Use an iterative formula, based on the equation in part **(b)**, to find the value of β correct to 3 significant figures. Give the result of each iteration to 5 significant figures. [3]

6 **(a)** Given that $\int_{-2a} \left(\frac{1}{2}e^{2x} + \frac{1}{4}e^{-x}\right)dx = 5$, where a is a positive constant, show that

$$a = \frac{1}{2}\ln\left(10 + \frac{1}{2}e^{-a} + \frac{1}{2}e^{-4a}\right).$$

[4]

(b) Hence show by calculation that the value of a lies between 1.0 and 1.2. [2]

(c) Use the iterative formula

$$a_{n+1} = \frac{1}{2}\ln\left(10 + \frac{1}{2}e^{-a_n} + \frac{1}{2}e^{-4a_n}\right)$$

to find the value of a correct to 4 significant figures. Give the result of each iteration to 6 significant figures. [3]

3 **(a)** Sketch, on a single diagram, the graphs of $y = 3e^{-2x}$ and $y = \sec x$ for values of x such that $0 \leqslant x < \frac{1}{2}\pi$. [2]

(b) Show that the x-coordinate of the point of intersection of the two graphs satisfies the equation $x = \frac{1}{2}\ln(3\cos x)$. [2]

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(c) Use an iterative formula, based on the equation in part **(b)**, to find the x-coordinate of the point of intersection correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

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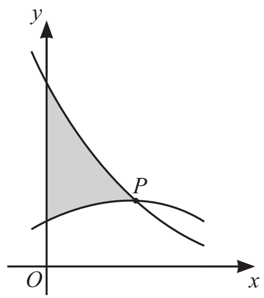

The diagram shows parts of the curves with equations $y = 4e^{-2x}$ and $y = 1 + 0.5\sin 3x$. Point P is a point of intersection of the curves, and the shaded region is bounded by the two curves and the y-axis.

(a) Show that the x-coordinate of P satisfies the equation $x = -0.5\ln(0.25 + 0.125\sin 3x)$. [1]

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(b) Use an iterative formula, based on the equation in part **(a)**, to find the x-coordinate of P correct to 4 significant figures. Use an initial value of 0.5 and give the result of each iteration to 6 significant figures. [3]

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(c) Hence find the area of the shaded region. Give your answer correct to 2 significant figures. [4]