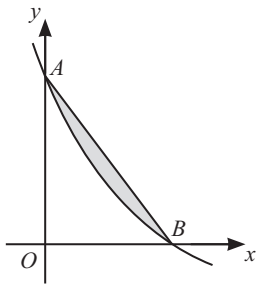
1 Find $\int 6\sin^2 x \, dx$. [3]

5

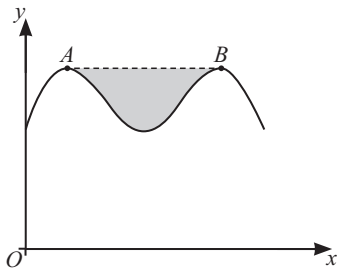


The diagram shows the curve with equation $y = 8e^{-\frac{1}{2}x} - 1$. The curve meets the axes at the points A and B. The shaded region is bounded by the curve and the line segment AB.

(a) Show that the x-coordinate of B is $6\ln 2$. [2]

(b) Find the area of the shaded region. Give your answer in the form $p\ln 2 - q$, where p and q are positive integers. [5]

5



The diagram shows the curve with equation $y = 4\cos 2x + 8\sin x$ for $0 \leqslant x \leqslant \pi$. The maximum points on the curve are denoted by A and B, and the shaded region is bounded by the line segment AB and the curve.

(a) Find the coordinates of A and B. [5]

(b) Find the exact area of the shaded region. [6]

6 **(a)** Given that $\int_{-2a}\left(\frac{1}{2}e^{2x}+\frac{1}{4}e^{-x}\right)dx = 5$, where a is a positive constant, show that

$$a = \tfrac{1}{2}\ln\left(10+\tfrac{1}{2}e^{-a}+\tfrac{1}{2}e^{-4a}\right).$$ [4]

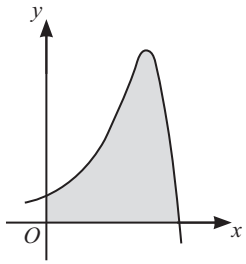
(b) Hence show by calculation that the value of a lies between 1.0 and 1.2. [2]

(c) Use the iterative formula

$$a_{n+1} = \tfrac{1}{2}\ln\left(10+\tfrac{1}{2}e^{-a_n}+\tfrac{1}{2}e^{-4a_n}\right)$$

to find the value of a correct to 4 significant figures. Give the result of each iteration to 6 significant figures. [3]

4



The diagram shows the curve with equation $y = 6e^{2x} - e^{3x}$. The shaded region is bounded by the axes and the curve.

(a) Find the exact x-coordinate of the maximum point. [3]

(b) Find the area of the shaded region. Give your answer in the form $\frac{p}{q}$, where p and q are integers. [4]

7 **(a)** Prove that $\sin^2 2x + 4\cos^2 x \cos 2x \equiv 4\cos^4 x$. [3]

(b) Find the set of possible values of the constant k for which the equation

$$\sin^2 2x + 4\cos^2 x \cos 2x + 5 = k$$

has no real solutions. [2]

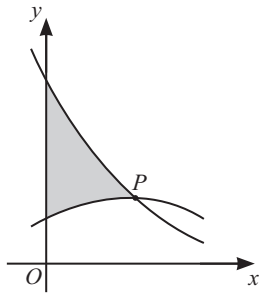
(c) Find the exact value of $\int_{-\frac{1}{3}\pi}^{\frac{1}{3}\pi} \sqrt{\sin^2 t + 4\cos^2\left(\frac{1}{2}t\right)\cos t}\,\mathrm{d}t$. [4]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

1 Show that $\int_2^8 \frac{1}{4x+1}\,dx = \ln a$, where a is an integer to be found. [3]

4



The diagram shows parts of the curves with equations $y = 4e^{-2x}$ and $y = 1 + 0.5\sin 3x$. Point P is a point of intersection of the curves, and the shaded region is bounded by the two curves and the y-axis.

(a) Show that the x-coordinate of P satisfies the equation $x = -0.5\ln(0.25 + 0.125\sin 3x)$. [1]

(b) Use an iterative formula, based on the equation in part **(a)**, to find the x-coordinate of P correct to 4 significant figures. Use an initial value of 0.5 and give the result of each iteration to 6 significant figures. [3]

(c) Hence find the area of the shaded region. Give your answer correct to 2 significant figures. [4]