8(a)	Integrate to obtain expression $\frac{1}{2}x^{\frac{3}{2}} \div \frac{3}{2}$
	Evaluate integral of the form $kx^{\frac{3}{2}}$ between limits 0 and 9 and subtract result from $\frac{27}{2}$
	Obtain $\frac{1}{3}x^{\frac{3}{2}}$ giving area under curve is 9 and shaded area is $\frac{9}{2}$
	Alternative Method for Question 8(a)
	Obtain $x = 4y^2$
	Integrate to obtain expression of form ky^3 and evaluate between limits 0 and $\frac{3}{2}$
	Obtain $\frac{4}{3}y^3$ and hence shaded area is $\frac{9}{2}$
8(b)	Attempt to express x^2 in terms of y
	Obtain or imply volume is $[\pi]\int 16y^4\,[\mathrm{d}y]$
	Integrate to obtain ky^5 and evaluate between limits 0 and $\frac{3}{2}$
	Obtain $\frac{16}{5}y^5$ and hence $\frac{243}{10}\pi$ or exact equivalent

11(a)	Substitute 4 to obtain gradient of curve is $\frac{1}{10}$
	Attempt equation of normal using $(4, 3)$ and $-\frac{1}{m}$ for their gradient
	$y = -10x + 43$
11(b)	$\left\{-16x^{-3}\right\}\left\{+40(2x-3)^{-3}\right\}$
	Substitute 4 to obtain $\frac{7}{100}$
11(c)	$[y =]\left\{-8x^{-1}\right\}\left\{+5(2x-3)^{-1}\right\}[+c]$
	Substitute $x = 4$, $y = 3$ in an integrated expression to find value of c
	Obtain $3 = -2 + 1 + c$ and hence $y = -8x^{-1} + 5(2x-3)^{-1} + 4$
	Substitute $x = -1$ to obtain $q = 11$

4(a)	$[k=]4$
4(b)	$\left\{\frac{k}{4}x^4\right\}+\left\{\frac{2x^{-2+1}}{-2+1}\right\}[+c]$
	$20=\frac{their\,k}{4}\times 2^4-\frac{2}{2}+c$
	$c=5$
	$[t=]4$

5(a)	$\left[\frac{dy}{dx}=\right]2x^{-\frac{1}{2}}-1$
	$2x^{-\frac{1}{2}}-1=0\Rightarrow x^{\frac{1}{2}}=2$
	$[a=]4$
	Alternative Method for Question 5(a):
	$y=4\sqrt{x}-(\sqrt{x})^2$
	Max at $\sqrt{x}=\frac{-b}{2a}=\frac{-4}{-2}=2$ or $-(\sqrt{x}-2)^2+4\quad\Rightarrow x^{\frac{1}{2}}=2$
	$[a=]4$

5(b)	$[x=\text{or}\,b=]\ 16$
	$\left[\int\left(4x^{\frac{1}{2}}-x\right)dx=\right]\left\{\frac{4}{\frac{3}{2}}x^{\frac{3}{2}}\right\}+\left\{-\frac{1}{2}x^2\right\}$
	[Area =] $\left(\frac{8}{3}\times16^{\frac{3}{2}}-\frac{1}{2}\times16^2\right)-\left(\frac{8}{3}\times4^{\frac{3}{2}}-\frac{1}{2}\times4^2\right)$
	$\left\{=\frac{128}{3}-\frac{40}{3}\right\}=\frac{88}{3}$ or $29\frac{1}{3}$

2(a)	[Gradient of tangent] = $4(2\times4-5)^3-9\times4^{\frac{1}{2}}\,[=90]$
	[Gradient of normal] = $-\frac{1}{90}$
2(b)	$[y]=\left\{\frac{1}{2}(2x-5)^4\right\}\left\{-6x^{\frac{3}{2}}\right\}[+c]$
	$-\frac{11}{2}=\frac{1}{2}\times(2\times4-5)^4-6\times4^{\frac{3}{2}}+c$
	$y=\frac{1}{2}(2x-5)^4-6x^{\frac{3}{2}}+2$

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4	Attempt to integrate $(x-16)-\left(5x^{\frac{3}{2}}-20x\right)\left[=-5x^{\frac{3}{2}}+21x-16\right]$
	$\left\{\frac{1}{2}x^2-16x\right\}[-]\left\{\left(2x^{\frac{5}{2}}-10x^2\right)\right\}\left[=-2x^{\frac{5}{2}}+\frac{21}{2}x^2-16x\right]$
	Use of limits 1 and 16
	391.5 or $\frac{783}{2}$

6	$6-3x=3 \Rightarrow x$-coordinate of point of intersection = 1
	$\left[\frac{18}{5}(5x+4)^{\frac{1}{2}}\right]_0^1$
	Area $=\frac{18}{5}\times3-\frac{18}{5}\times2\left[=\frac{18}{5}\right]$
	Line crosses x-axis at x = 2, so triangle area $=\frac{1}{2}\times3\times1$
	Total area $=\frac{51}{10}$

9(a)	$\left[\dfrac{dy}{dx}=\right]\ \dfrac{-24x^{-2}}{-2}\ \left[=\dfrac{12}{x^2}\right][+c]$
	$\dfrac{12}{(-2)^2}+c=0\ \ [\Rightarrow\ c=-3]$
	$\left[\dfrac{dy}{dx}=\right]\ \dfrac{12}{x^2}-3$ or $12x^{-2}-3$

9(b)	$\left[\dfrac{12}{x^2}-3=0\ \Rightarrow\ x^2=4\ \Rightarrow\ x=\pm 2\ \Rightarrow\right]\ x=2$
	$\left[\dfrac{d^2y}{dx^2}=\right]-\dfrac{24}{(2)^3}$ or -3 or $<0\ \Rightarrow$ maximum
9(c)	$[y=]-12x^{-1}+\mathit{their}\,cx\left[=-\dfrac{12}{x}-3x\right][+d]$
	$19=-\dfrac{12}{-2}-3\times(-2)+d\ \ [\Rightarrow d=7]$
	$y=-\dfrac{12}{x}-3x+7$

9(d)	$\frac{12}{x^2}-3=-\frac{9}{4} \Rightarrow \frac{12}{x^2}=\frac{3}{4} \Rightarrow x=\ldots \left[\Rightarrow x^2=16 \Rightarrow x=4, y=-8\right]$
	Gradient of normal = $\frac{4}{9}$
	$\frac{y+8}{(x-4)}=\frac{4}{9}$
	$4x-9y-88=0$