

|       |                                                                               |
|-------|-------------------------------------------------------------------------------|
| 11(a) | Substitute 4 to obtain gradient of curve is $\frac{1}{10}$                    |
|       | Attempt equation of normal using (4, 3) and $-\frac{1}{m}$ for their gradient |
|       | $y = -10x + 43$                                                               |
|       |                                                                               |
| 11(b) | $\{-16x^{-3}\}\{+40(2x-3)^{-3}\}$                                             |
|       | Substitute 4 to obtain $\frac{7}{100}$                                        |
|       |                                                                               |
| 11(c) | $[y = ]\{-8x^{-1}\}\{+5(2x-3)^{-1}\}[+c]$                                     |
|       | Substitute $x=4, y=3$ in an integrated expression to find value of $c$        |
|       | Obtain $3 = -2 + 1 + c$ and hence $y = -8x^{-1} + 5(2x-3)^{-1} + 4$           |
|       | Substitute $x=-1$ to obtain $q=11$                                            |
|       |                                                                               |

|      |                                                                                                                     |
|------|---------------------------------------------------------------------------------------------------------------------|
| 4(a) | $[k = ]4$                                                                                                           |
|      |                                                                                                                     |
| 4(b) | $\left\{\frac{k}{4}x^4\right\} + \left\{\frac{2x^{-2+1}}{-2+1}\right\} [+c]$                                        |
|      | $20 = \frac{\text{their } k}{4} \times 2^4 - \frac{2}{2} + c$                                                       |
|      | $c = 5$                                                                                                             |
|      | $[t = ]4$                                                                                                           |
|      |                                                                                                                     |
| 5(a) | $\left[\frac{dy}{dx} = \right] 2x^{-\frac{1}{2}} - 1$                                                               |
|      | $2x^{-\frac{1}{2}} - 1 = 0 \Rightarrow x^{\frac{1}{2}} = 2$                                                         |
|      |                                                                                                                     |
|      | $[a = ]4$                                                                                                           |
|      | <b>Alternative Method for Question 5(a):</b>                                                                        |
|      | $y = 4\sqrt{x} - (\sqrt{x})^2$                                                                                      |
|      | Max at $\sqrt{x} = \frac{-b}{2a} = \frac{-4}{-2} = 2$<br>or $-(\sqrt{x} - 2)^2 + 4 \Rightarrow x^{\frac{1}{2}} = 2$ |
|      | $[a = ]4$                                                                                                           |
|      |                                                                                                                     |

|      |                                                                                                                                                                             |
|------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 5(b) | $[x = \text{or } b =] 16$                                                                                                                                                   |
|      | $\left[ \int (4x^{\frac{1}{2}} - x) dx = \right] \left\{ \frac{4}{\frac{3}{2}} x^{\frac{3}{2}} \right\} + \left\{ -\frac{1}{2} x^2 \right\}$                                |
|      | $[\text{Area} =] \left( \frac{8}{3} \times 16^{\frac{3}{2}} - \frac{1}{2} \times 16^2 \right) - \left( \frac{8}{3} \times 4^{\frac{3}{2}} - \frac{1}{2} \times 4^2 \right)$ |
|      | $\left\{ = \frac{128}{3} - \frac{40}{3} \right\} = \frac{88}{3} \text{ or } 29\frac{1}{3}$                                                                                  |

|      |                                                                                                                                                                   |
|------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 9(a) | $\frac{4 \times 3 \times (-2)}{(3x-6)^3} + \frac{1 \times 3 \times (-3)}{(3x-6)^4}$<br>or $4 \times 3 \times (-2)(3x-6)^{-3} + 1 \times 3 \times (-3)(3x-6)^{-4}$ |
|      | Decreasing.                                                                                                                                                       |
|      |                                                                                                                                                                   |
| 9(b) | $f^{-1}$ exists because $f$ is a decreasing function<br>Or $f^{-1}$ exists because it is one-to-one, or passes the horizontal line test.                          |
|      |                                                                                                                                                                   |
| 9(c) | $g(x) > 4a - 3$                                                                                                                                                   |
|      |                                                                                                                                                                   |
| 9(d) | <b>Either</b> $4a - 3 > 2$ allow with $x$ or $a$ or $\geq$                                                                                                        |
|      | $a > \frac{5}{4}$ or $a \geq \frac{5}{4}$ oe                                                                                                                      |
|      | <b>Or</b> $3 \times (4a - 3) - 6 > 0$ allow with $x$ or $a$ or $\geq$                                                                                             |
|      | $a > \frac{5}{4}$ or $a \geq \frac{5}{4}$ oe                                                                                                                      |
|      |                                                                                                                                                                   |

|      |                                                                                        |
|------|----------------------------------------------------------------------------------------|
| 2(a) | $[\text{Gradient of tangent}] = 4(2 \times 4 - 5)^3 - 9 \times 4^{\frac{1}{2}} [= 90]$ |
|      | $[\text{Gradient of normal}] = -\frac{1}{90}$                                          |
|      |                                                                                        |
| 2(b) | $[y] = \left\{ \frac{1}{2}(2x-5)^4 \right\} \left\{ -6x^{\frac{3}{2}} \right\} [+c]$   |
|      | $-\frac{11}{2} = \frac{1}{2} \times (2 \times 4 - 5)^4 - 6 \times 4^{\frac{3}{2}} + c$ |
|      | $y = \frac{1}{2}(2x-5)^4 - 6x^{\frac{3}{2}} + 2$                                       |
|      |                                                                                        |

|      |                                                                                                         |
|------|---------------------------------------------------------------------------------------------------------|
| 7(a) | $\left[ \frac{dy}{dx} \right] 8x - \frac{18}{x^3}$                                                      |
|      | At $x = 2, \frac{dy}{dx} = \frac{55}{4}$                                                                |
|      | $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} \Rightarrow \frac{55}{4} = -5 \times \frac{dt}{dx}$ |
|      | $\left[ \frac{dx}{dt} \right] = -\frac{4}{11}$                                                          |
|      |                                                                                                         |

|      |                                                                        |
|------|------------------------------------------------------------------------|
| 7(b) | $8x - \frac{18}{x^3} = 0 \left[ \Rightarrow x^4 = \frac{9}{4} \right]$ |
|      | $x = \pm \sqrt{\frac{3}{2}} \text{ or } \pm \frac{\sqrt{6}}{2}$        |
|      | $y = 4 \text{ (for both)}$                                             |
|      | $\left[ \frac{d^2y}{dx^2} \right] = 8 + \frac{54}{x^4}$                |
|      | So both are minima                                                     |
|      |                                                                        |

|      |                                                                                                                                                                   |
|------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 4(a) | $\left[ \frac{dy}{dx} \right] = \frac{3}{2}ax^{\frac{1}{2}} - 12$                                                                                                 |
|      | $\left[ \frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt} = \right] \left( \frac{3}{2}a \times 9^{\frac{1}{2}} - 12 \right) \times 5$                            |
|      | $\left[ \frac{dy}{dt} = \right] 5 \left( \frac{9}{2}a - 12 \right)$ or $\frac{45}{2}a - 60$ or $22.5a - 60$ or $\frac{45a - 120}{2}$<br>or $\frac{15}{2}(3a - 8)$ |
|      |                                                                                                                                                                   |
| 4(b) | $\frac{3}{2}a \times \left( \frac{1}{4} \right)^{\frac{1}{2}} - 12 = 0$                                                                                           |
|      | $\Rightarrow a = 16$                                                                                                                                              |
|      |                                                                                                                                                                   |

|      |                                                                                            |
|------|--------------------------------------------------------------------------------------------|
| 9(a) | $\left[ \frac{dy}{dx} = \right] \frac{-24x^{-2}}{-2} \left[ = \frac{12}{x^2} \right] [+c]$ |
|      | $\frac{12}{(-2)^2} + c = 0 \quad [\Rightarrow c = -3]$                                     |
|      | $\left[ \frac{dy}{dx} = \right] \frac{12}{x^2} - 3$ or $12x^{-2} - 3$                      |
|      |                                                                                            |

|      |                                                                                                     |
|------|-----------------------------------------------------------------------------------------------------|
| 9(b) | $\left[ \frac{12}{x^2} - 3 = 0 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2 \Rightarrow \right] x = 2$ |
|      | $\left[ \frac{d^2y}{dx^2} = \right] -\frac{24}{(2)^3}$ or $-3$ or $< 0 \Rightarrow$ maximum         |
|      |                                                                                                     |
| 9(c) | $[y =] -12x^{-1} + \text{their } cx \left[ = -\frac{12}{x} - 3x \right] [+d]$                       |
|      | $19 = -\frac{12}{-2} - 3 \times (-2) + d \quad [\Rightarrow d = 7]$                                 |
|      | $y = -\frac{12}{x} - 3x + 7$                                                                        |
|      |                                                                                                     |

|      |                                                                                                                                                                  |
|------|------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 9(d) | $\frac{12}{x^2} - 3 = -\frac{9}{4} \Rightarrow \frac{12}{x^2} = \frac{3}{4} \Rightarrow x = \dots \left[ \Rightarrow x^2 = 16 \Rightarrow x = 4, y = -8 \right]$ |
|      | Gradient of normal = $\frac{4}{9}$                                                                                                                               |
|      | $\frac{y+8}{(x-4)} = \frac{4}{9}$                                                                                                                                |
|      | $4x - 9y - 88 = 0$                                                                                                                                               |
|      |                                                                                                                                                                  |