Week 34

A-Level Mathematics

Homework bundle for this week

本周作业合集

exercises.lol

Contents 目录

A-Level Mathematics Name: _____ Score: _____

1. The coefficient of friction is 0.4 and the normal reaction is 100 N. What is the maximum (limiting) friction force, in N?

   _____

2. A car starts from rest and accelerates at 2.5 m/s^2 for 4 s. Using $v = u + at$, what is its speed (m/s)?

   _____

3. A 2 kg body moving at 3 m/s hits a stationary 1 kg body and they stick together. What is their common speed (m/s)?

   _____

4. A resultant force gives a 5 kg mass an acceleration of 2 m/s^2. What is the force ($F = ma$), in N?

   _____

5. What is the kinetic energy of a 2 kg object moving at 3 m/s ($KE = \frac{1}{2}mv^2$), in J?

   _____

6. A particle is in equilibrium when the forces on it:

   ☐ have a vector sum of zero

   ☐ are all pointing up

   ☐ are all equal in size

   ☐ are all friction

7. Momentum is defined as:

   ☐ mass × velocity

   ☐ mass × acceleration

   ☐ force × distance

   ☐ mass / velocity

8. The horizontal component of a force F at angle θ to the horizontal is:

   ☐ $F \cos \theta$

   ☐ $F \sin \theta$

   ☐ $F \tan \theta$

   ☐ F / θ

9. On a velocity–time graph, the area under the line gives the:

   ☐ displacement

   ☐ acceleration

   ☐ force

   ☐ mass

10. In a collision, the total momentum of the two bodies is conserved.

    ☐ True ☐ False

A-Level Mathematics

1. **40**

   Maximum friction = μR = 0.4 × 100 = 40 N.

2. **10**

   v = 0 + 2.5 × 4 = 10 m/s.

3. **2**

   2(3) + 1(0) = (2+1)v, so 6 = 3v, giving v = 2 m/s.

4. **10**

   F = ma = 5 × 2 = 10 N.

5. **9**

   KE = ½ × 2 × 3^2 = ½ × 2 × 9 = 9 J.

6. **have a vector sum of zero**

   Equilibrium means the forces balance — their vector sum is zero.

7. **mass × velocity**

   Momentum = mass × velocity, a vector quantity.

8. **F cos θ**

   Horizontal component = F cos θ; vertical component = F sin θ.

9. **displacement**

   Area = displacement; the gradient gives the acceleration.

10. **True**

    Total momentum before equals total momentum after the collision.

A-LEVEL MATHEMATICS • EXERCISE SHEET

4.1 Forces and equilibrium

Name: _____ Class: _____ Date: _____ **Total: 14 marks**

KEY WORDS friction 摩擦 • coefficient of friction 摩擦系数 • equilibrium 平衡 • forces and equilibrium 力与平衡 • limiting friction 最大静摩擦力

Objective

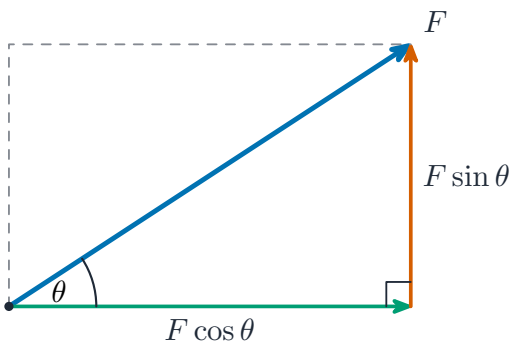
Build the skills to answer exam questions on **forces and equilibrium** 力与平衡 — resolving forces into **components** 分量, the condition for **equilibrium** 平衡, and **friction** 摩擦 ($F \leqslant \mu R$). Take $g = 10$ m s^{-2}.

You must be able to:

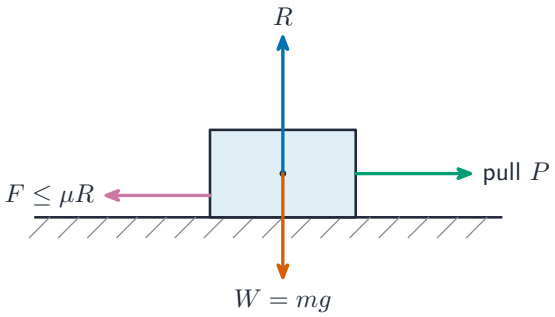
- resolve a force into horizontal and vertical components
- use "components in any direction sum to zero" for equilibrium
- use $F \leqslant \mu R$ and limiting friction $F = \mu R$

1 Worked examples

■ Resolving & friction



A force at angle θ has parts $F\cos\theta$ (horizontal) and $F\sin\theta$ (vertical)



The contact force splits into the normal reaction R and friction $F \leqslant \mu R$

A 5 kg block rests on a rough horizontal floor ($\mu = 0.3$). Max friction $= \mu R = 0.3 \times 50 = 15$ N — a horizontal pull below 15 N will not move it.

2 Practice

2.1 A force of 20 N acts at 60° above the horizontal. Find its horizontal component. [2]

2.2 A block of weight 40 N rests on a rough surface, $\mu = 0.25$. Find the maximum friction available. [2]

3 Exam-style questions

3.1 A particle is in equilibrium under three forces. The sum of the forces is: [1]

- **A** maximum
- **B** zero
- **C** equal to the weight
- **D** equal to μR

3.2 A box of mass 8 kg rests on a rough horizontal floor. A horizontal force of 30 N is applied but the box does not move.

(a) Find the normal reaction. [1]

(b) Given the box is on the point of slipping, find the coefficient of friction. [3]

3.3 A particle of weight W hangs in equilibrium from two light strings making angles 30° and 60° with the horizontal. The tensions are T_1 and T_2. Given $W = 20$ N, find T_1 and T_2. [5]

Solutions

2.1 $20\cos 60° = 20 \times 0.5 = 10$ N.

2.2 $F_{\max} = \mu R = 0.25 \times 40 = 10$ N.

3.1 **B**. In equilibrium the resultant (vector sum) of the forces is zero.

3.2 (a) $R = mg = 8 \times 10 = 80$ N.
(b) on the point of slipping $F = \mu R$ and $F = 30$ N; $\mu = \dfrac{30}{80}$; $= 0.375$.

3.3 resolve: horizontal $T_2\cos 60° = T_1\cos 30°$; vertical $T_1\sin 30° + T_2\sin 60° = 20$; from the first $T_2(0.5) = T_1(0.866) \Rightarrow T_2 = 1.732\,T_1$; substitute: $0.5T_1 + 0.866(1.732T_1) = 20 \Rightarrow 2T_1 = 20 \Rightarrow T_1 = 10$ N, $T_2 = 17.3$ N.

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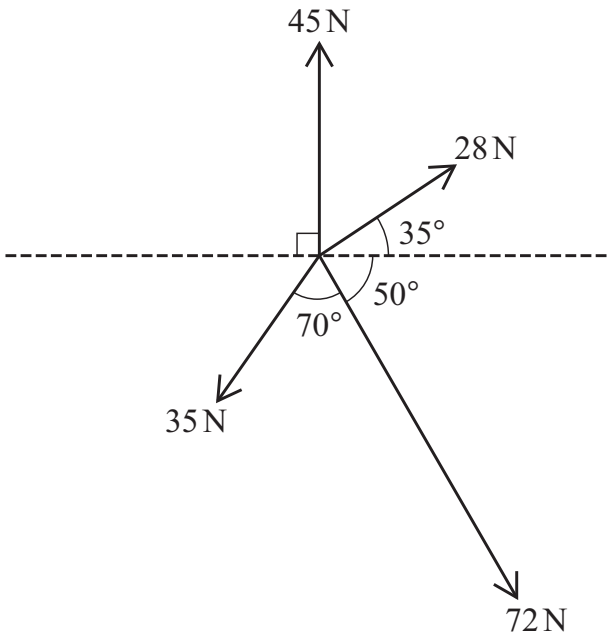

Coplanar forces of magnitudes 45 N, 28 N, 72 N and 35 N act at a point in the directions shown in the diagram.

Find, in either order, the magnitude and direction of the resultant force. [6]

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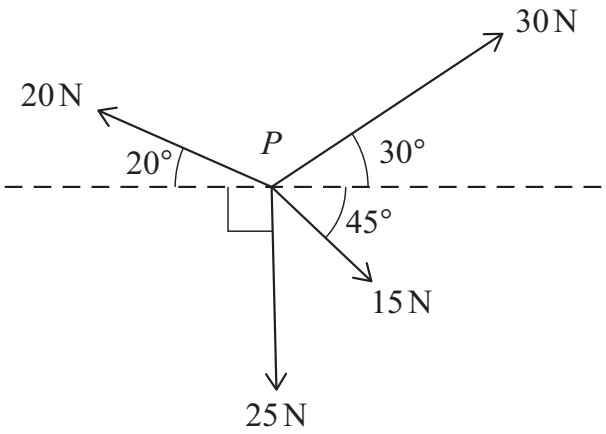

Fig. 7.1

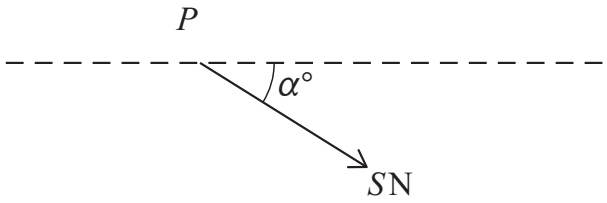

Fig. 7.2

Four coplanar forces of magnitudes 20 N, 30 N, 15 N and 25 N act at a point P in the directions shown in Fig. 7.1. The forces act in a vertical plane. The resultant of these forces has magnitude S N and acts at an angle $\alpha°$ below the horizontal as shown in Fig. 7.2.

(a) Find the value of S and the value of α. [6]

A small ring of mass 0.6 kg is threaded on a rough straight horizontal wire. The four forces shown in Fig. 7.1 act on the ring and are in the same vertical plane as the wire. The ring starts from rest and takes 3 s to travel a distance of 2 m along the wire.

(b) Find the coefficient of friction between the ring and the wire. [5]

Additional page

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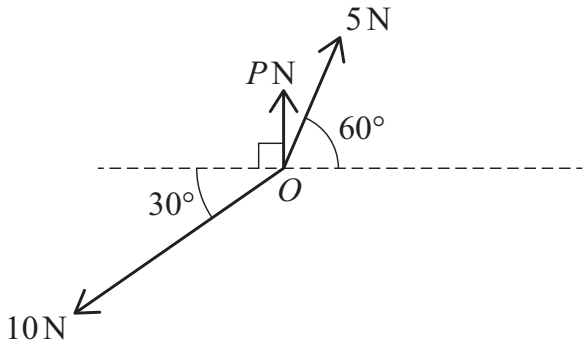


Three coplanar forces of magnitudes PN, 5 N and 10 N act at a point O, as shown in the diagram. The resultant of the three forces has magnitude QN and acts in a direction perpendicular to the force of magnitude PN.

Find the value of P and the value of Q. [4]

A-LEVEL MATHEMATICS • EXERCISE SHEET

4.2 Kinematics of motion in a straight line

Name: _____ Class: _____ Date: _____ **Total: 16 marks**

KEY WORDS distance 距离 • velocity 速度 • speed 速率 • displacement 位移 • constant acceleration 匀加速

Objective

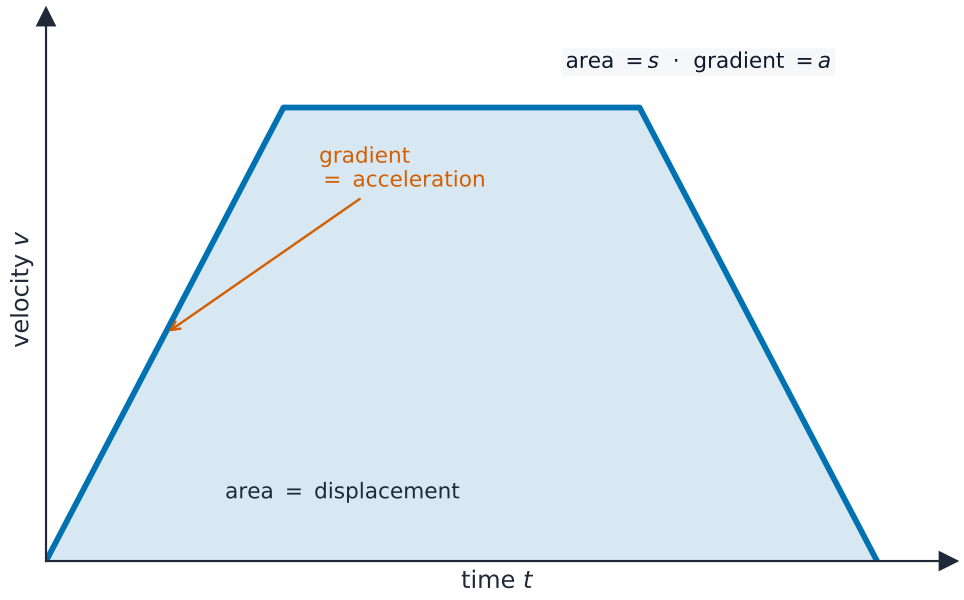
Build the skills to answer exam questions on **kinematics** 运动学 — the **velocity–time graph** 速度时间图, the **suvat** equations for constant acceleration, and using calculus for variable acceleration. Take $g = 10$ m s^{-2}.

You must be able to:

- read displacement (area) and acceleration (gradient) from a v–t graph
- use $v = u + at$, $s = ut + \frac{1}{2}at^2$, $v^2 = u^2 + 2as$
- differentiate/integrate between s, v, a for variable acceleration

1 Worked examples

■ Graph & suvat



Area under a v–t graph = displacement; gradient = acceleration

Car from rest, $a = 2.5$ m s^{-2} for 4 s: $v = 2.5 \times 4 = 10$ m s^{-1}; $s = \frac{1}{2}(2.5)(16) = 20$

m.

Variable: if $s = t^3 - 3t^2$ then $v = \dfrac{ds}{dt} = 3t^2 - 6t$, $a = \dfrac{dv}{dt} = 6t - 6$.

2 Practice

2.1 A particle accelerates from 4 m s^{-1} at 3 m s^{-2} for 5 s. Find its final velocity. [2]

2.2 A v–t graph is a straight line from $(0, 2)$ to $(6, 14)$. Find the acceleration. [2]

3 Exam-style questions

3.1 A stone is dropped from rest. After 3 s its speed is ($g = 10$): [1]

- **A** 10 m s^{-1}
- **B** 20 m s^{-1}
- **C** 30 m s^{-1}
- **D** 45 m s^{-1}

3.2 A car travelling at 30 m s^{-1} brakes with constant deceleration and stops in 75 m.

(a) Find the deceleration. [3]

(b) Find the time taken to stop. [2]

3.3 A particle moves so that its velocity is $v = 12t - 3t^2$ m s^{-1} at time t s.

(a) Find the time when the particle is instantaneously at rest (other than $t = 0$). [2]

(b) Find the distance travelled in the first 4 seconds. [4]

Solutions

2.1 $v = u + at = 4 + 3(5) = 19$ m s^{-1}.

2.2 gradient $= \dfrac{14-2}{6-0} = 2$ m s^{-2}.

3.1 C. $v = u + gt = 0 + 10(3) = 30$ m s^{-1}.

3.2 (a) $v^2 = u^2 + 2as$: $0 = 30^2 + 2a(75)$; $a = -\dfrac{900}{150} = -6$, so deceleration 6 m s^{-2}.
(b) $v = u + at$: $0 = 30 - 6t \Rightarrow t = 5$ s.

3.3 (a) $v = 0$: $12t - 3t^2 = 0 \Rightarrow 3t(4-t) = 0 \Rightarrow t = 4$ s.
(b) $v \geqslant 0$ for $0 \leqslant t \leqslant 4$, so distance $= \displaystyle\int_0^4 (12t - 3t^2)\,dt = [6t^2 - t^3]_0^4; = 96 - 64 = 32$ m.

7 A particle P starts from a point O and moves in a straight line. The velocity $v\,\text{m s}^{-1}$ of P, at time t s after leaving O, is given by $v = \frac{1}{3}(2t-3)(t-4)$.

(a) Find the acceleration of P when $t = 2$. [2]

(b) Find the total distance travelled by P in the first 3 seconds of its motion. [5]

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(c) Determine whether P returns to O. [2]

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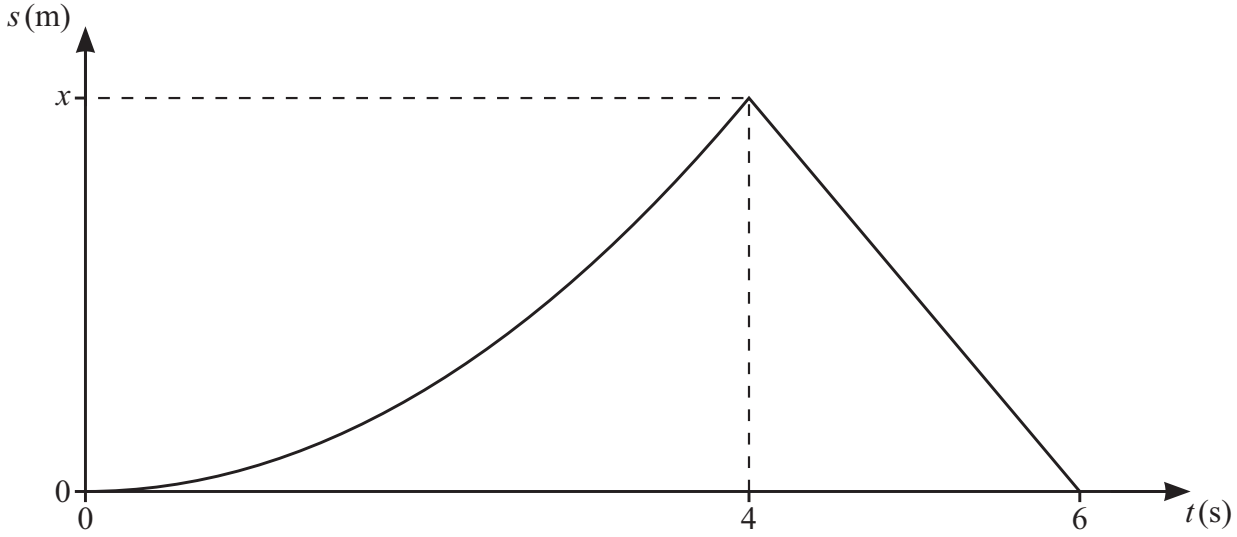

The diagram shows the displacement-time graph for the motion of a particle. The particle starts from rest at a point O and travels with constant acceleration $a\,\text{m s}^{-2}$, taking 4 s to move a distance of x m. The particle then returns to O with constant speed of $40\,\text{m s}^{-1}$, over a period of 2 s.

Find the value of x and the value of a. [3]

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3 A car of mass 1400 kg travelling on a straight horizontal road accelerates uniformly from a speed of $15\,\text{m s}^{-1}$ to $20\,\text{m s}^{-1}$ over a distance of 175 m.

(a) Find the time taken to travel the 175 m. [2]

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There is a constant resistance force of 800 N to the motion of the car.

(b) (i) Use an energy method throughout to find the average power of the car's engine as the speed increases from $15\,\text{m s}^{-1}$ to $20\,\text{m s}^{-1}$. [4]

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(ii) Find the steady speed that the car could maintain on the horizontal road if the engine is working at the power found in **(b)(i)**. [2]

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4 A particle P of mass 0.1 kg is projected vertically upwards with speed $30\,\text{m}\,\text{s}^{-1}$ from horizontal ground. At the same instant a particle Q of mass 0.4 kg is projected vertically upwards with speed $10\,\text{m}\,\text{s}^{-1}$ from a height of 15 m above the ground. P and Q move in the same vertical line.

(a) Find the height above the ground at which P and Q collide. [4]

When P and Q collide, they coalesce.

(b) Find the speed of the combined particle at the instant that it reaches the ground. [5]

6 A particle starts from rest at a point O. The acceleration of the particle at time t s after leaving O is a m s^{-2}, where

$$a = 2(t+1)^{-\frac{1}{2}} - 1 \text{ for } t \geqslant 0.$$

Find the distance that the particle travels from O until the time at which its acceleration is zero. [8]

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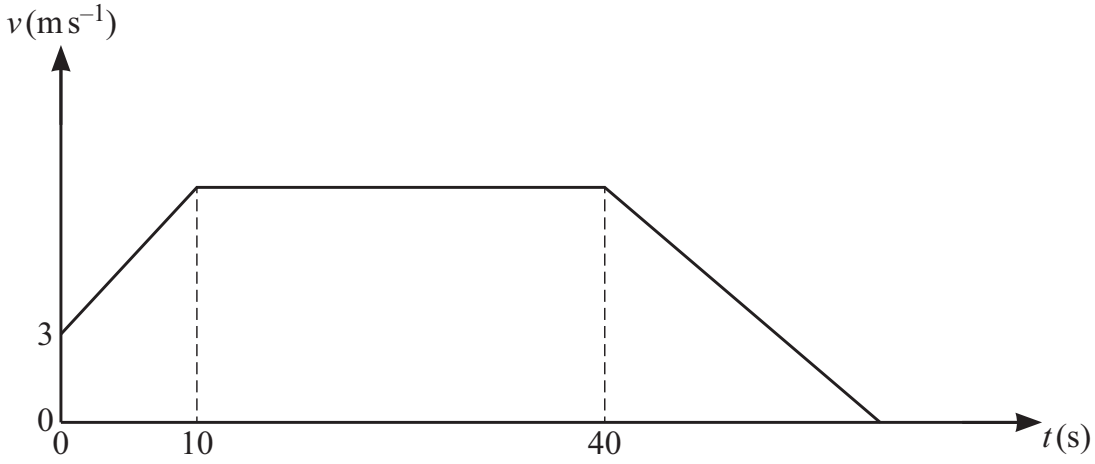

The diagram shows the velocity-time graph of the motion of a cyclist. The graph consists of three straight line segments. The cyclist passes a point O with speed 3 m s^{-1} and then accelerates for 10 s with constant acceleration 0.5 m s^{-2}. He then travels at constant speed for 30 s before decelerating, coming to rest at point P, covering a distance of 80 m whilst decelerating.

(a) Find the total time taken for the journey from O to P. [3]

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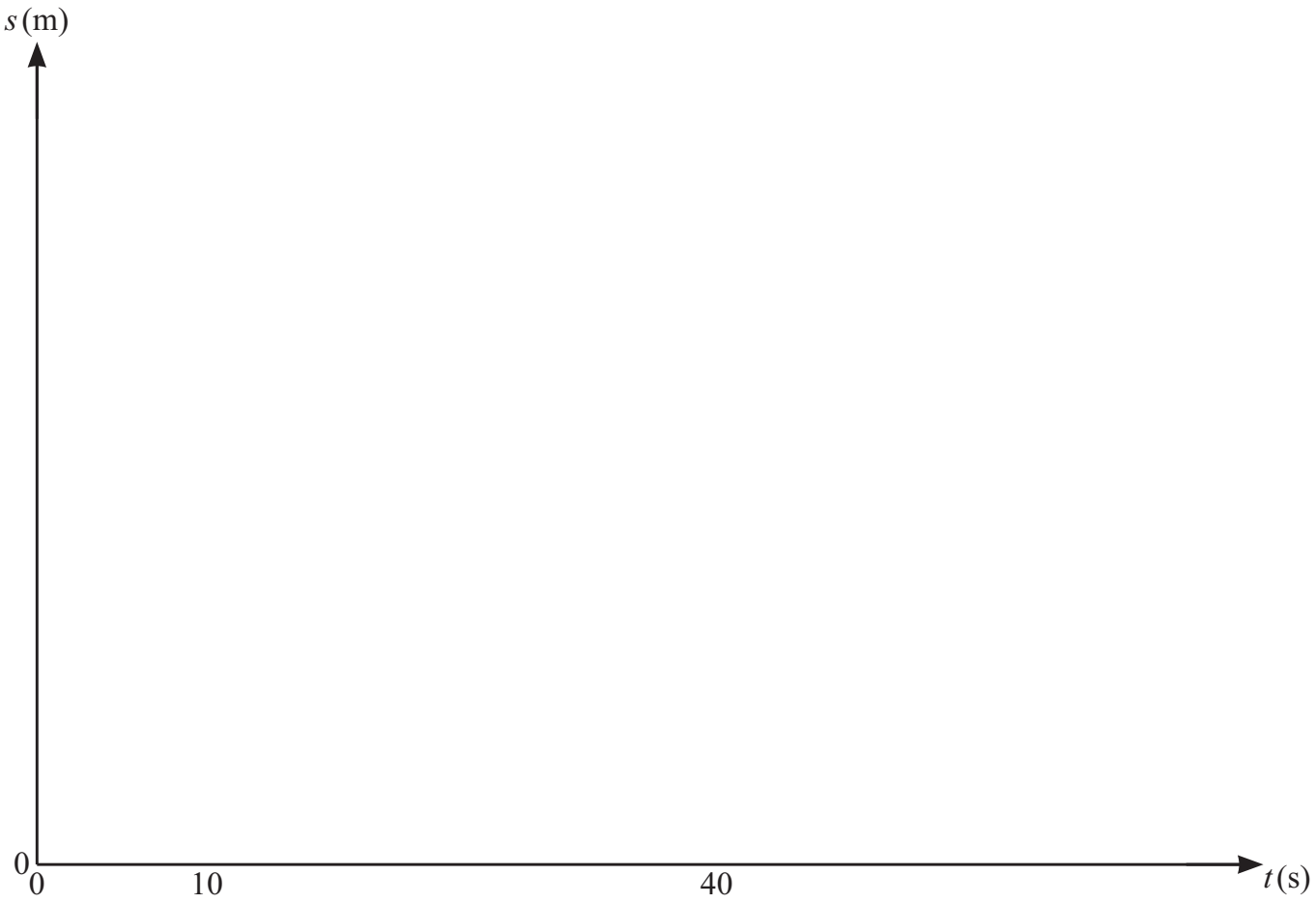
(b) On the given axes, sketch a displacement-time graph for the cyclist's journey from O to P, showing on your graph the distances travelled after 10 s and 40 s. [4]



7 A particle X moves along a straight track, starting from a point O at time $t = 0$. The displacement of X from O at time t s is s m, where $s = 3t^{\frac{3}{2}} - 6t$.

(a) Find the time at which X is instantaneously at rest, and hence find the total distance travelled by X between $t = 0$ and $t = 16$. [6]

A second particle Y moves along another straight track, starting from a point P at time $t = 0$. The acceleration of Y at time t s is $a\,\text{m s}^{-2}$, where $a = 0.8 - 0.6t$. The velocity of Y when it leaves P is $7.5\,\text{m s}^{-1}$.

(b) When the velocity of Y is $-9.6\,\text{m s}^{-1}$, show that the displacement of X from O is equal to the displacement of Y from P. [7]

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A-LEVEL MATHEMATICS • EXERCISE SHEET

4.3 Momentum

Name: _____ Class: _____ Date: _____ **Total: 10 marks**

KEY WORDS momentum 动量 • conservation of momentum 动量守恒 • mass 质量 • speed 速率 • velocity 速度

Objective

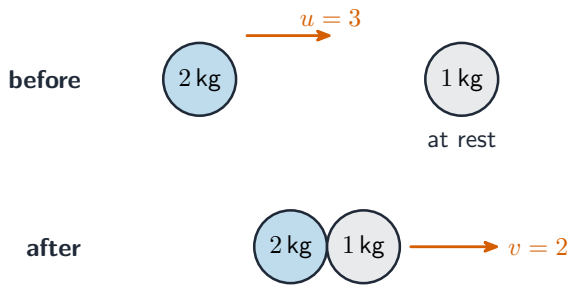
Build the skills to answer exam questions on **momentum** 动量 — linear momentum and the **conservation of momentum** 动量守恒 in collisions. Take $g = 10$ m s^{-2}.

You must be able to:

- calculate momentum (mass × velocity), treating direction with signs
- apply $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$
- handle "stick together" (coalesce) collisions

1 Worked examples

■ Conservation of momentum



$$2(3) + 1(0) = (2 + 1)\,v$$

Total momentum is the same before and after a collision

2 kg at 3 m s^{-1} hits a stationary 1 kg and they stick:

$$2(3) + 1(0) = (2 + 1)v \Rightarrow v = \tfrac{6}{3} = 2 \text{ m s}^{-1}.$$

Sign convention: choose a positive direction; a velocity the other way is negative.

2 Practice

2.1 Find the momentum of a 0.5 kg ball moving at 8 m s^{-1}. [1]

2.2 State what physical quantity is conserved in a collision with no external horizontal force. [1]

3 Exam-style questions

3.1 A 3 kg body at 4 m s^{-1} collides with a 2 kg body at rest and they coalesce. Their common speed is: [1]

- **A** 1.2 m s^{-1}
- **B** 2.0 m s^{-1}
- **C** 2.4 m s^{-1}
- **D** 4.0 m s^{-1}

3.2 A particle A of mass 0.4 kg moving at 5 m s^{-1} collides directly with particle B of mass 0.6 kg moving in the **opposite** direction at 2 m s^{-1}. After the collision they move together.

(a) Find their common velocity, stating its direction. [4]

3.3 A 2 kg ball moving at 6 m s^{-1} strikes a 4 kg ball at rest. After the collision the 2 kg ball continues in the same direction at 1 m s^{-1}. Find the speed of the 4 kg ball. [3]

Solutions

2.1 $p = mv = 0.5 \times 8 = 4$ kg m s^{-1}.

2.2 (linear) momentum.

3.1 C. $3(4) = (3+2)v \Rightarrow v = \dfrac{12}{5} = 2.4$ m s^{-1}.

3.2 take A's direction positive: $0.4(5)+0.6(-2) = (0.4+0.6)v$; $2-1.2 = 1.0\,v \Rightarrow v = 0.8$; 0.8 m s^{-1} in A's original direction.

3.3 $2(6) + 4(0) = 2(1) + 4v$; $12 = 2 + 4v \Rightarrow v = \dfrac{10}{4} = 2.5$ m s^{-1}.

2 Two particles, P and Q, of masses 3 kg and 5 kg respectively, are at rest on a smooth horizontal plane. P is projected at a speed of $4\,\text{m s}^{-1}$ directly towards Q. After P and Q collide, P has speed $1\,\text{m s}^{-1}$.

(a) Find the two possible speeds of Q after the collision. [3]

It is given that λ J of kinetic energy, where $\lambda > 0$, is lost during the collision.

(b) Find the value of λ. [2]

6 A particle A of mass 2.5 kg is released from rest from the top of a smooth plane, which makes an angle of $\sin^{-1} 0.2$ with the horizontal. The particle A collides 2 seconds later with a particle B, of mass 3 kg, which is moving up a line of greatest slope of the plane. The speed of B immediately before the collision is $3.5\,\text{m s}^{-1}$. Immediately after the collision, B has a velocity of $0.5\,\text{m s}^{-1}$ down the plane.

Find the distance A moves up the plane after the collision. [6]

4 A particle P of mass 0.1 kg is projected vertically upwards with speed $30\,\text{m}\,\text{s}^{-1}$ from horizontal ground. At the same instant a particle Q of mass 0.4 kg is projected vertically upwards with speed $10\,\text{m}\,\text{s}^{-1}$ from a height of 15 m above the ground. P and Q move in the same vertical line.

(a) Find the height above the ground at which P and Q collide. [4]

When P and Q collide, they coalesce.

(b) Find the speed of the combined particle at the instant that it reaches the ground. [5]

5 When a particle P of mass m kg has speed $u\,\text{m s}^{-1}$, its momentum is 4 N s and its kinetic energy is 16 J.

(a) Find the value of m and the value of u. [3]

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P is now projected on a smooth horizontal surface with speed $v\,\text{m s}^{-1}$ directly towards a particle Q of mass 1.25 kg which is stationary. After P and Q collide, the velocity of P is $w\,\text{m s}^{-1}$ and the velocity of Q is $2w\,\text{m s}^{-1}$. The loss of kinetic energy in the collision is 25 J.

(b) Find the value of v and the value of w. [4]

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A-LEVEL MATHEMATICS • EXERCISE SHEET

4.4 Newton's laws of motion

Name: _____ Class: _____ Date: _____ **Total: 15 marks**

KEY WORDS connected particles 连接质点 • newton's laws 牛顿运动定律 • thrust 推力 • mass 质量 • acceleration 加速度

Objective

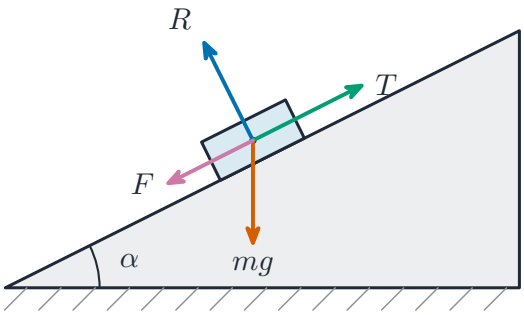
Build the skills to answer exam questions on **Newton's laws** 牛顿运动定律 — $F = ma$, weight $W = mg$, motion on an **inclined plane** 斜面, and **connected particles** 连接质点. Take $g = 10$ m s^{-2}.

You must be able to:

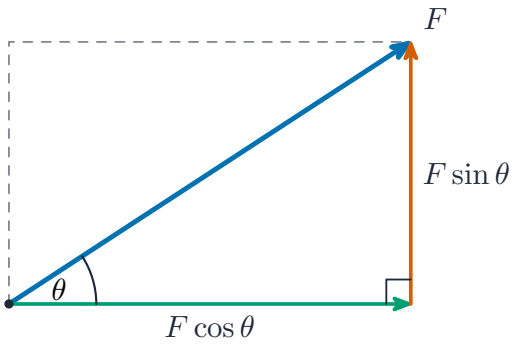
- apply $F = ma$ with weight, tension, friction and thrust
- resolve along and perpendicular to a slope
- analyse connected particles (string over a pulley)

1 Worked examples

■ Inclined plane



On a slope, resolve forces along the slope and at right angles to it, then use $F = ma$ along the slope



Resolving a force into perpendicular components

12 kg block pulled up a 20° rough slope ($\mu = 0.4$), $a = 2$ m s^{-2}: $R = 120\cos 20° = 112.8$ N; $F = \mu R = 45.1$ N. Up the slope: $T - mg\sin 20° - F = ma$, so $T = 24 + 41.0 + 45.1 = 110$ N.

2 Practice

2.1 A resultant force of 18 N acts on a 3 kg mass. Find the acceleration. [2]

2.2 Find the weight of a 7 kg object ($g = 10$). [1]

3 Exam-style questions

3.1 A 5 kg mass is pulled along a smooth horizontal floor by a horizontal force of 20 N. Its acceleration is: [1]

- **A** 2 m s^{-2}
- **B** 4 m s^{-2}
- **C** 5 m s^{-2}
- **D** 25 m s^{-2}

3.2 A block of mass 4 kg is released from rest on a smooth plane inclined at 30° to the horizontal.

(a) Find the acceleration down the slope. [3]

(b) Find the speed after it has slid 5 m. [2]

3.3 Two particles of masses 3 kg and 5 kg are joined by a light inextensible string over a smooth pulley, and released from rest.

(a) Find the acceleration of the system. [4]

(b) Find the tension in the string. [2]

Solutions

2.1 $a = \dfrac{F}{m} = \dfrac{18}{3} = 6 \text{ m s}^{-2}$.

2.2 $W = mg = 7 \times 10 = 70$ N.

3.1 B. $a = \dfrac{20}{5} = 4 \text{ m s}^{-2}$.

3.2 (a) along the slope: $mg \sin 30° = ma \Rightarrow a = g \sin 30° = 10(0.5) = 5 \text{ m s}^{-2}$.
(b) $v^2 = u^2 + 2as = 0 + 2(5)(5) = 50 \Rightarrow v = 7.07 \text{ m s}^{-1}$.

3.3 (a) for the system, $5g - 3g = (5 + 3)a$; $20 = 8a \Rightarrow a = 2.5 \text{ m s}^{-2}$.
(b) for the 3 kg mass: $T - 3g = 3a \Rightarrow T = 30 + 3(2.5) = 37.5$ N.

1 A car of mass 900 kg is moving along a straight horizontal road against a constant resistance to motion of 350 N. At an instant when the car is moving at $15\,\text{m s}^{-1}$ its acceleration is $0.25\,\text{m s}^{-2}$.

(a) Find the driving force of the car's engine at this instant. [2]

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(b) Find the power of the car's engine at this instant. [2]

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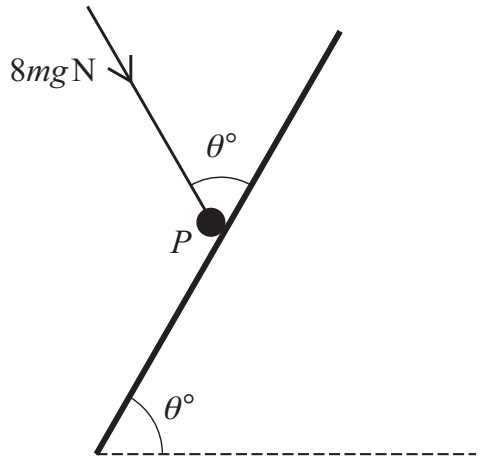

A particle P of mass m kg is in equilibrium on a rough plane inclined at an angle $\theta°$ to the horizontal. The equilibrium of P is maintained by a force of magnitude $8mg$ N making an angle $\theta°$ with a line of greatest slope (see diagram). The coefficient of friction between P and the plane is 0.5 and P is on the point of slipping down the plane.

Find the value of θ. [6]

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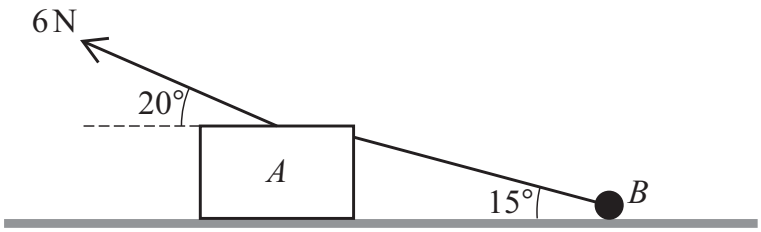

A block A of mass 2 kg and a particle B of mass 0.5 kg are connected by a light inextensible string inclined at 15° to the horizontal. They are pulled across a horizontal surface with acceleration $1.2\,\text{m s}^{-2}$ by a force of magnitude 6 N, applied to A, acting at 20° above the horizontal as shown in the diagram. The string and the force applied to A are in the same vertical plane.

The contact between B and the surface is smooth and the contact between A and the surface is rough.

(a) Find the tension in the string. [2]

(b) Find the coefficient of friction between A and the surface. [6]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

2 A railway locomotive of mass 240 000 kg is towing a coach of mass 36 000 kg down a hill inclined at an angle of $\sin^{-1} 0.04$ to the horizontal. The driving force produced by the locomotive is 450 000 N and there are resistances to motion of 120 000 N on the locomotive and 15 000 N on the coach. The coupling between the locomotive and the coach is light, rigid and parallel to the hill.

Find the acceleration of the locomotive and the tension in the coupling. [5]

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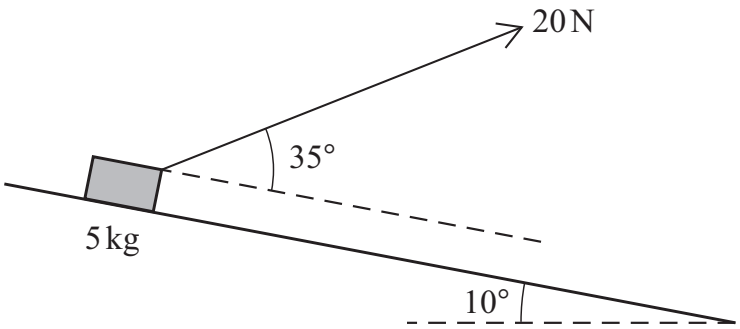

A block of mass 5 kg is being pulled straight down a line of greatest slope of a rough plane by a force of magnitude 20 N. The plane is inclined at an angle of 10° to the horizontal and the 20 N force acts at an angle of 35° above the line of greatest slope of the plane (see diagram). The coefficient of friction between the block and the plane is 0.4. The speed of the block when it passes a point O is $2\,\text{m}\,\text{s}^{-1}$.

Find the speed of the block when it has moved 3 m down the plane from O. [6]

1 A block of mass 12 kg is being pulled by a rope up a rough plane. The plane is inclined at an angle of 20° above the horizontal. The rope pulling the block is parallel to a line of greatest slope of the plane. The coefficient of friction between the block and the plane is 0.4. The acceleration of the block is $2\,\text{m}\,\text{s}^{-2}$.

Find the tension in the rope. [4]

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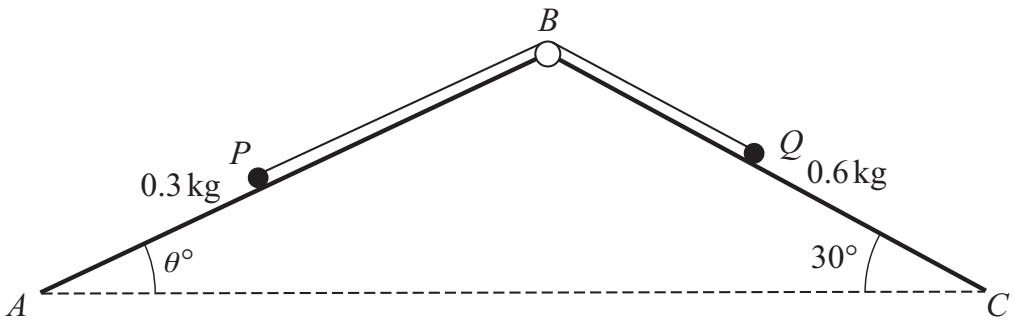

Two particles, P and Q, of masses 0.3 kg and 0.6 kg respectively, are attached to the ends of a light inextensible string. The string passes over a smooth pulley fixed at a point B where the inclined planes AB and BC meet. P lies on the smooth plane AB which is inclined at an angle $\theta°$ to the horizontal where $\sin\theta° = 0.4$. Q lies on the plane BC which is inclined at 30° to the horizontal. The string is taut and the particles can move on lines of greatest slope of the two planes (see diagram). The particles are released from rest.

(a) It is given that the plane BC is smooth.

Find the tension in the string and the acceleration of Q. [5]

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(b) It is given instead that the plane BC is rough. The work done against the frictional force when Q moves 2 m down the plane is 1.8 J. You should assume that P does **not** reach the pulley and that Q does **not** reach C.

Use an energy method to find the speed of Q when it has moved 2 m down the plane. [4]

A-LEVEL MATHEMATICS • EXERCISE SHEET

4.5 Energy, work and power

Name: _____ Class: _____ Date: _____ **Total: 14 marks**

KEY WORDS power 功率 • work done 功 • kinetic 动能 • potential energy 势能 • mass 质量

Objective

Build the skills to answer exam questions on **energy, work and power** 能量、功与功率 — **work done** 功, **kinetic** 动能 and **potential energy** 势能, the **work–energy** principle, and **power** 功率 $P = Fv$. Take $g = 10$ m s^{-2}.

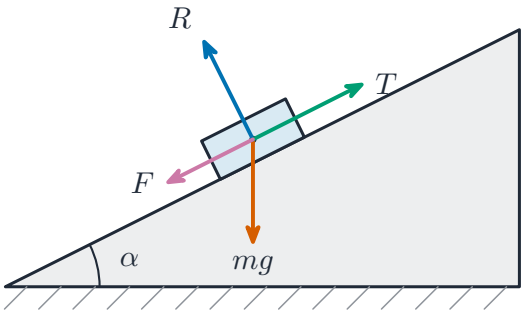
You must be able to:

- calculate work $W = Fd\cos\theta$, KE $= \frac{1}{2}mv^2$, PE $= mgh$
- use the work–energy principle (work done = change in energy)
- use $P = Fv$ for power

1 Worked examples

Energy & power

A roller coaster trades potential energy for kinetic energy as it rises and falls



Work and energy on an inclined plane

Engine 12 **kW, speed** 20 **m s^{-1} on the level:** $P = Fv \Rightarrow F = \dfrac{12000}{20} = 600$ N.

Work–energy: (driving force work) − (resistance work) = change in KE.

2 Practice

2.1 Find the kinetic energy of a 2 kg body moving at 6 m s^{-1}. [2]

2.2 A force of 50 N moves a body 4 m in its own direction. Find the work done. [1]

3 Exam-style questions

3.1 A 1500 W motor raises a load at constant speed. In 10 s it does work of: [1]

- **A** 150 J
- **B** 1500 J
- **C** 15000 J
- **D** 150000 J

3.2 A car of mass 1200 kg accelerates from 10 m s^{-1} to 20 m s^{-1} on a level road.

(a) Find the increase in kinetic energy. [3]

(b) If a constant resistance of 400 N acts over the 150 m taken, find the total work done by the engine. [3]

3.3 A cyclist and bicycle of total mass 90 kg freewheel from rest down a slope, dropping a vertical height of 20 m. Resistances do 9000 J of work.

(a) Using energy, find the speed at the bottom. [4]

Solutions

2.1 KE $= \frac{1}{2}mv^2 = \frac{1}{2}(2)(36) = 36$ J.

2.2 $W = Fd = 50 \times 4 = 200$ J.

3.1 **C**. work = power $\times$ time $= 1500 \times 10 = 15000$ J.

3.2 (a) ΔKE $= \frac{1}{2}(1200)(20^2 - 10^2) = 600(400 - 100) = 600(300) = 180000$ J.
(b) work by engine = ΔKE+ work against resistance $= 180000 + 400(150)$; $= 180000 + 60000 = 240000$ J.

3.3 (a) energy: mgh − (resistance work) $= \frac{1}{2}mv^2$; $90(10)(20) - 9000 = \frac{1}{2}(90)v^2$; $18000 - 9000 = 45v^2 \Rightarrow v^2 = 200$; $v = 14.1$ m s^{-1}.

1 A car of mass 900 kg is moving along a straight horizontal road against a constant resistance to motion of 350 N. At an instant when the car is moving at $15\,\text{m s}^{-1}$ its acceleration is $0.25\,\text{m s}^{-2}$.

(a) Find the driving force of the car's engine at this instant. [2]

(b) Find the power of the car's engine at this instant. [2]

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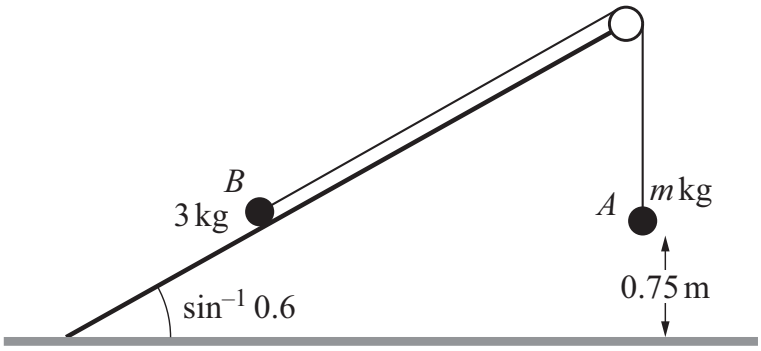

Two particles, A and B, of masses m kg and 3 kg respectively, are connected by a light inextensible string. Particle B is on a fixed plane which is at an angle of $\sin^{-1} 0.6$ to the horizontal ground. The string passes over a fixed smooth pulley at the top of the plane. Particle A hangs vertically below the pulley and is 0.75 m above the ground (see diagram).

The system is released from rest. In the subsequent motion B moves up a line of greatest slope of the plane and does not reach the pulley. As B moves up the plane there is a constant resistance to its motion of magnitude $\frac{10}{3}$ N. The speed of A immediately before it hits the ground is $2\,\text{m}\,\text{s}^{-1}$.

Use an energy method to find the value of m. [6]

3 A car of mass 1400 kg travelling on a straight horizontal road accelerates uniformly from a speed of $15\,\text{m s}^{-1}$ to $20\,\text{m s}^{-1}$ over a distance of 175 m.

(a) Find the time taken to travel the 175 m. [2]

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There is a constant resistance force of 800 N to the motion of the car.

(b) (i) Use an energy method throughout to find the average power of the car's engine as the speed increases from $15\,\text{m s}^{-1}$ to $20\,\text{m s}^{-1}$. [4]

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(ii) Find the steady speed that the car could maintain on the horizontal road if the engine is working at the power found in **(b)(i)**. [2]

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4 A lorry of mass 18 000 kg is travelling along a straight road.

(a) On a horizontal section of the road, the power of the lorry's engine is constant. There is a constant resistance to motion of 1600 N.

(i) The steady speed which the lorry can maintain with the engine working at power P W is $30\,\text{m s}^{-1}$.

Find the value of P. [1]

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(ii) At an instant when the speed of the lorry is $16\,\text{m s}^{-1}$, its engine is working at a power of 40 kW.

Find the acceleration of the lorry at this instant. [2]

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(b) When the lorry has reached a speed of $20\,\text{m s}^{-1}$, it begins to ascend a section of road inclined at an angle $\alpha°$ to the horizontal. The engine now works at a power of 120 kW. There is no change in the lorry's speed as it ascends the hill. The constant resistance to motion remains 1600 N.

Find the value of α. [3]

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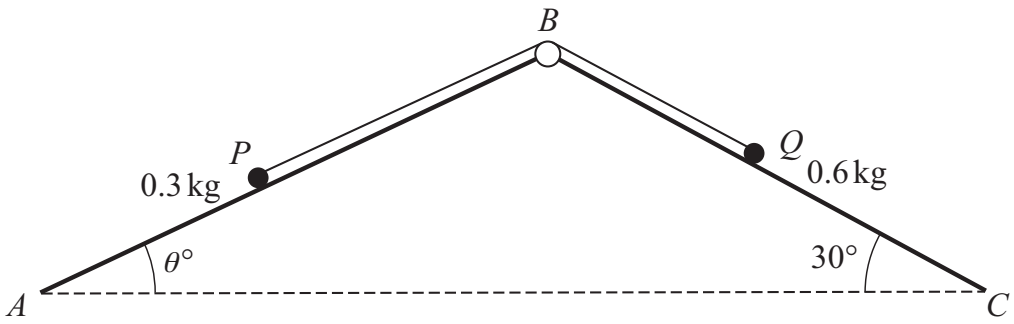

Two particles, P and Q, of masses 0.3 kg and 0.6 kg respectively, are attached to the ends of a light inextensible string. The string passes over a smooth pulley fixed at a point B where the inclined planes AB and BC meet. P lies on the smooth plane AB which is inclined at an angle $\theta°$ to the horizontal where $\sin\theta° = 0.4$. Q lies on the plane BC which is inclined at 30° to the horizontal. The string is taut and the particles can move on lines of greatest slope of the two planes (see diagram). The particles are released from rest.

(a) It is given that the plane BC is smooth.

Find the tension in the string and the acceleration of Q. [5]

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(b) It is given instead that the plane BC is rough. The work done against the frictional force when Q moves 2 m down the plane is 1.8 J. You should assume that P does **not** reach the pulley and that Q does **not** reach C.

Use an energy method to find the speed of Q when it has moved 2 m down the plane. [4]