A-LEVEL MATHEMATICS • HOLIDAY HOMEWORK 假期作业

1 Pure Mathematics 1 – Part 1

Name: _____ Class: _____ Date: _____

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)		
factorising	因式分解			
parabola	抛物线			
quadratic	二次式			
coefficients	系数			
Completing the square	配方			
vertex	顶点			
discriminant	判别式			
real roots	实根			

Recall

At IGCSE you worked with quadratics. Bring this with you:

- You solved quadratic equations by **factorising** 因式分解.
- You drew the **parabola** 抛物线 — the U-shaped curve of $y = ax^2 + bx + c$.

At AS Pure 1 we add a tidy form that reveals the curve's turning point, and a quick test for how many solutions an equation has.

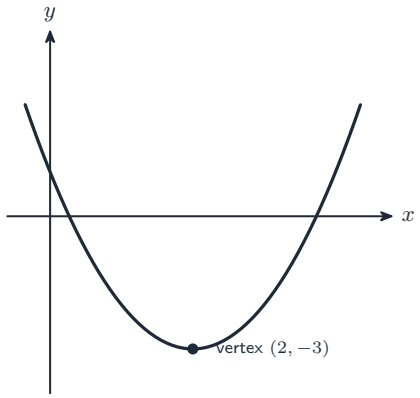
New ideas

1 Completing the square

A **quadratic** 二次式 has the form ax^2+bx+c; the numbers a, b, c are its **coefficients** 系数.

Completing the square 配方 rewrites it as $a(x + p)^2 + q$. This form shows the **vertex** 顶点 (turning point) of the curve at $(-p,\ q)$.

Worked example. $x^2 - 4x + 1 = (x - 2)^2 - 4 + 1 = (x - 2)^2 - 3$. The vertex is at $(2, -3)$.

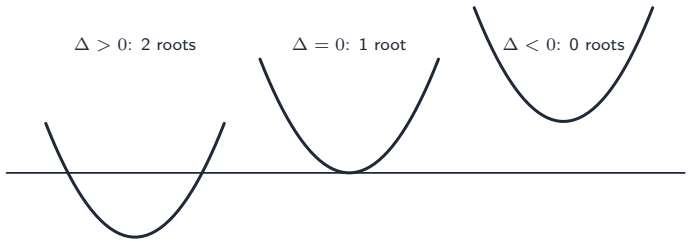


Micro-check. Write $x^2 - 6x$ in the form $(x + p)^2 + q$. [1]

2 The discriminant

The **discriminant** 判别式 of $ax^2 + bx + c$ is $\Delta = b^2 - 4ac$. It tells you how many **real roots** 实根 (real solutions) the equation $ax^2 + bx + c = 0$ has:

- $b^2 - 4ac > 0$ — two real roots;
- $b^2 - 4ac = 0$ — one (repeated) real root;
- $b^2 - 4ac < 0$ — no real roots.



Worked example. For $x^2 - 4x + 1$: $\Delta = (-4)^2 - 4(1)(1) = 16 - 4 = 12 > 0$, so there are **two** real roots.

Micro-check. Find the discriminant of $x^2 + 2x + 5$, and state the number of real roots. [2]

Watch out: to complete the square, halve b and square it. In $a(x+p)^2+q$ the vertex is at $(-p,\ q)$ — note the *sign flip* on p. For the discriminant: $b^2 - 4ac > 0 \to$ two roots, $= 0 \to$ one, $< 0 \to$ none.

Practice

1.1 Write $x^2 - 8x + 3$ in the form $(x + p)^2 + q$. [2]

1.2 State the vertex of the curve $y = (x - 3)^2 - 4$. [1]

1.3 Find the discriminant of $2x^2 + 3x + 1$, and state how many real roots the equation $2x^2 + 3x + 1 = 0$ has. [2]

1.4 State the number of real roots of $x^2 - 4x + 4 = 0$ (use the discriminant). [2]

1.5 Solve $x^2 - 5x + 6 = 0$ by factorising. [2]

1.6 Challenge. Find the values of k for which $x^2 + kx + 9 = 0$ has exactly one (repeated) real root. [3]