

6.3 Continuous random variables

Name: _____ Class: _____ Date: _____ **Total: 17 marks**

KEY WORDS probability density function 概率密度函数 • median 中位数
• continuous random variable 连续型随机变量 • cumulative distribution function 累积分布函数
• percentiles 百分位数

Objective

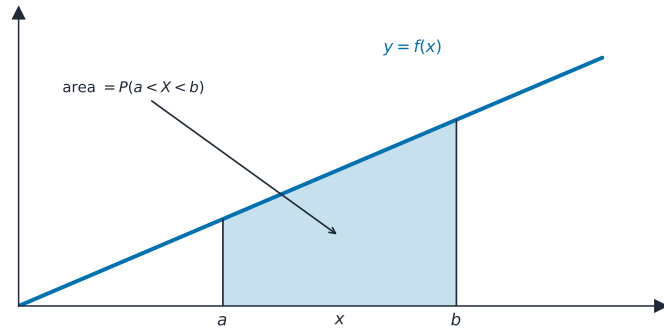
Build the skills to answer exam questions on **continuous random variables** 连续型随机变量 — the **probability density function** 概率密度函数 $f(x)$, finding probabilities and $E(X)$, and the **cumulative distribution function** 累积分布函数 $F(x)$.

You must be able to:

- use $\int f(x) dx = 1$ to find a constant; find $P(a < X < b) = \int_a^b f dx$
- find $E(X) = \int x f(x) dx$ (and $\text{Var}(X)$)
- find $F(x)$ and use it for the median/percentiles

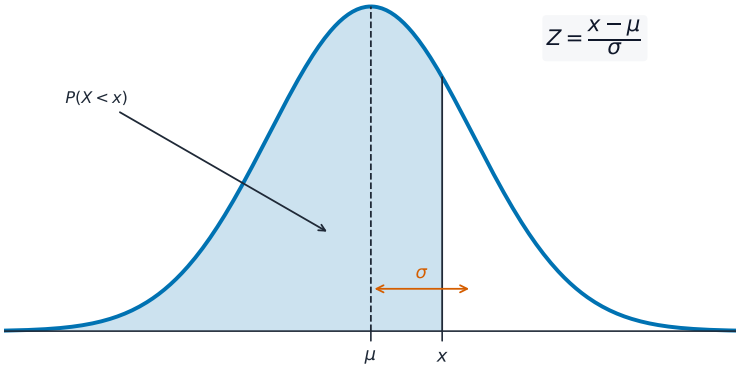
1 Worked examples

■ Density and expectation



$$\int_{-\infty}^{\infty} f = 1 \quad \cdot \quad P(a < X < b) = \int_a^b f$$

$P(a < X < b)$ is the area under $f(x)$ between a and b ; the total area is 1



The normal distribution is a continuous random variable

$$f(x) = \frac{1}{2}x \text{ for } 0 \leq x \leq 2: E(X) = \int_0^2 x \cdot \frac{1}{2}x dx = \left[\frac{x^3}{6} \right]_0^2 = \frac{8}{6} = \frac{4}{3}.$$

Median m : solve $F(m) = 0.5$.

2 Practice

2.1 $f(x) = kx$ for $0 \leq x \leq 2$. Show that $k = \frac{1}{2}$. [2]

2.2 For $f(x) = \frac{1}{2}x$ on $[0, 2]$, find $P(X < 1)$. [2]

3 Exam-style questions

3.1 For a valid pdf, $\int_{-\infty}^{\infty} f(x) dx$ must equal: [1]

- | | |
|---|---|
| A | B |
| C | D |
- A** 0
B 0.5
C 1
D ∞

3.2 A continuous random variable has $f(x) = \frac{3}{4}(1-x^2)$ for $-1 \leq x \leq 1$ (and 0 elsewhere).

(a) Verify that $\int_{-1}^1 f(x) dx = 1$. [3]

(b) Find $P(X > 0.5)$. [3]

3.3 A variable has $f(x) = \frac{3}{8}x^2$ for $0 \leq x \leq 2$.

(a) Find $E(X)$. [3]

(b) Find the median m . [3]
