

Solutions

Each answer shows the steps, then the **result**.

2.1

$$\bullet \int_0^2 kx \, dx = \left[\frac{kx^2}{2} \right]_0^2 = 2k = 1 \Rightarrow k = \frac{1}{2}$$

2.2

$$\bullet \int_0^1 \frac{1}{2}x \, dx = \left[\frac{x^2}{4} \right]_0^1 = \frac{1}{4}$$

3.1 C

- The total area under a pdf is 1

3.2

(a)

$$\bullet \int_{-1}^1 \frac{3}{4}(1-x^2) \, dx = \frac{3}{4} \left[x - \frac{x^3}{3} \right]_{-1}^1 = \frac{3}{4} \left(\frac{2}{3} - \left(-\frac{2}{3}\right) \right) = \frac{3}{4} \cdot \frac{4}{3} = 1$$

(b)

$$\bullet \int_{0.5}^1 \frac{3}{4}(1-x^2) \, dx = \frac{3}{4} \left[x - \frac{x^3}{3} \right]_{0.5}^1 = \frac{3}{4} \left(\frac{2}{3} - (0.5 - 0.0417) \right) = \frac{3}{4}(0.6667 - 0.4583) = 0.156$$

3.3

(a)

$$\bullet E(X) = \int_0^2 x \cdot \frac{3}{8}x^2 \, dx = \frac{3}{8} \left[\frac{x^4}{4} \right]_0^2 = \frac{3}{8} \cdot 4 = \frac{3}{2}$$

(b)

$$\bullet F(m) = \int_0^m \frac{3}{8}x^2 \, dx = \frac{m^3}{8} = 0.5 \Rightarrow m^3 = 4 \Rightarrow m = 1.59$$