

5.4 Discrete random variables

Name: _____ Class: _____ Date: _____ **Total: 12 marks**

KEY WORDS geometric 几何分布 • discrete random variable 离散型随机变量
• binomial 二项分布 • variance 方差 • expectation 期望

Objective

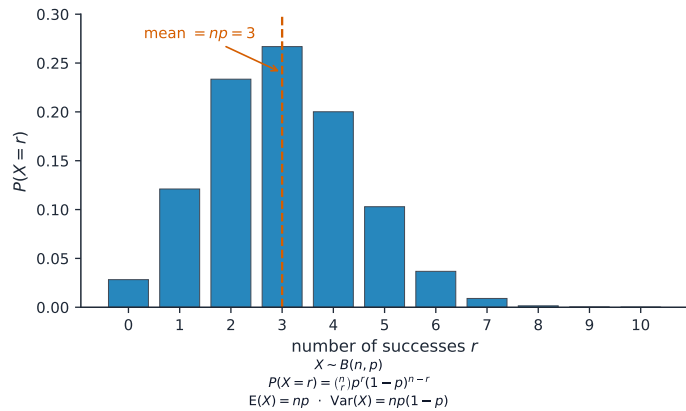
Build the skills to answer exam questions on **discrete random variables** 离散型随机变量 — probability distributions, **expectation** 期望 and **variance** 方差, the **binomial** 二项分布 and **geometric** 几何分布 distributions.

You must be able to:

- use a distribution table (probabilities sum to 1); find $E(X)$ and $\text{Var}(X)$
- use $B(n, p)$: $P(X = r) = \binom{n}{r} p^r (1 - p)^{n-r}$, $E(X) = np$
- use the geometric model $P(X = r) = (1 - p)^{r-1} p$, $E(X) = \frac{1}{p}$

1 Worked examples

■ Binomial distribution



$B(10, 0.3)$: each bar is $P(X = r)$, clustered around the mean $np = 3$

$X \sim B(10, 0.3)$: $P(X = 2) = \binom{10}{2} (0.3)^2 (0.7)^8 = 0.233$; $E(X) = np = 3$.

Distribution: $E(X) = \sum xP(X=x)$, $\text{Var}(X) = \sum x^2P(X=x) - [E(X)]^2$.

2 Practice

2.1 $X \sim B(8, 0.25)$. Find $E(X)$. [1]

2.2 A geometric variable has $p = 0.2$. Find $E(X)$. [1]

3 Exam-style questions

3.1 For $X \sim B(12, 0.5)$, the variance $\text{Var}(X)$ is: [1]

- A

B

C

D
- A 6
B 3
C 12
D 1.5

3.2 A discrete random variable X has the distribution:

x	1	2	3	4
$P(X = x)$	0.1	0.3	k	0.2

(a) Find k . [1]

(b) Find $E(X)$ and $\text{Var}(X)$. [5]

3.3 A fair coin is tossed 6 times. Find the probability of obtaining exactly 4 heads. [3]
