

4.1 Forces and equilibrium

Name: _____ Class: _____ Date: _____

Total: 32 marks

KEY WORDS friction 摩擦 • equilibrium 平衡 • coefficient of friction 摩擦系数
 • limiting equilibrium 极限平衡 • forces and equilibrium 力与平衡

Objective

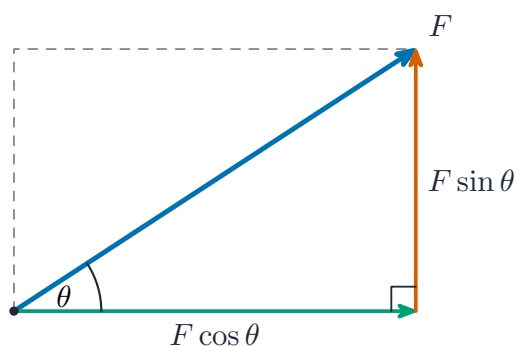
Build the skills to answer exam questions on **forces and equilibrium** 力与平衡 — resolving forces into **components** 分量, the condition for **equilibrium** 平衡, and **friction** 摩擦 ($F \leq \mu R$). Take $g = 10 \text{ m s}^{-2}$.

You must be able to:

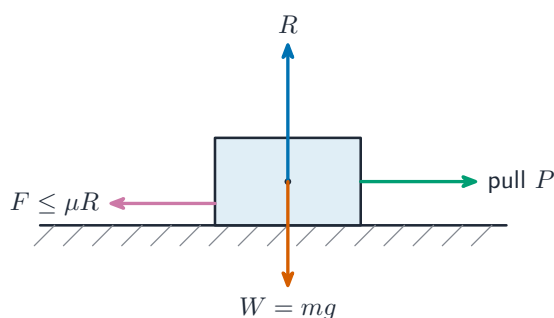
- resolve a force into horizontal and vertical components
- use “components in any direction sum to zero” for equilibrium
- use $F \leq \mu R$ and limiting friction $F = \mu R$

1 Worked examples

■ Resolving & friction



A force at angle θ has parts $F \cos \theta$ (horizontal) and $F \sin \theta$ (vertical)



The contact force splits into the normal reaction R and friction $F \leq \mu R$

A 5 kg block rests on a rough horizontal floor ($\mu = 0.3$). Max friction $= \mu R = 0.3 \times 50 = 15 \text{ N}$ — a horizontal pull below 15 N will not move it.

■ Resolving, and the two conditions for equilibrium

A force at an angle is replaced by two perpendicular **components**: along a chosen direction it is $F \cos \theta$, perpendicular to it $F \sin \theta$, where θ is measured **from that direction**. Getting the cosine and sine the right way round is the single commonest error; check by asking whether the component should vanish when $\theta = 90^\circ$.

Equilibrium means the resultant is zero, which gives two equations:

$$\sum F_x = 0 \quad \sum F_y = 0$$

Choose the axes to kill a force. On an inclined plane, resolving **along** and **perpendicular to** the slope removes the need to resolve the normal contact force and the friction at all — only the weight has to be split, into $mg \sin \alpha$ down the slope and $mg \cos \alpha$ into it.

Friction. $F \leq \mu R$, with **equality only when the object is on the point of sliding** ("limiting equilibrium"). Writing $F = \mu R$ for an object that is merely stationary is wrong, and it is a mark.

Worked example. A box of mass 8 kg rests on a rough plane inclined at 20° , held by a rope along the slope. The coefficient of friction is 0.3 and the box is on the point of sliding **down**. Find the tension.

1. Take axes along and perpendicular to the slope; $g = 9.81$.
2. Perpendicular: $R = mg \cos 20^\circ = 8(9.81) \cos 20^\circ = 73.8 \text{ N}$.
3. On the point of sliding down, so friction acts **up** the slope at its maximum:
 $F = \mu R = 0.3(73.8) = 22.1 \text{ N}$.
4. Along the slope: $T + F = mg \sin 20^\circ$, so $T = 8(9.81) \sin 20^\circ - 22.1 = 26.8 - 22.1 = 4.7 \text{ N}$.

△ **Friction opposes the motion that would happen**, so its direction depends on which way the object is about to slide. If the box were about to slide **up**, friction

would act down the slope and the tension would be $26.8 + 22.1 = 48.9$ N — the same numbers, a completely different answer.

2 Practice

2.1 A force of 20 N acts at 60° above the horizontal. Find its horizontal component. [2]

2.2 A block of weight 40 N rests on a rough surface, $\mu = 0.25$. Find the maximum friction available. [2]

2.3 A force of 20 N acts at 35° above the horizontal. Find its horizontal and vertical components. [2]

2.4 A block of mass 5 kg rests on a rough horizontal table with $\mu = 0.4$. Find the least horizontal force that would move it. [3]

2.5 State the condition under which $F = \mu R$ rather than $F \leq \mu R$. [1]

3 Exam-style questions

3.1 A particle is in equilibrium under three forces. The sum of the forces is: [1]

A	B
C	D

A maximum

B zero

C equal to the weight

D equal to μR

3.2 A box of mass 8 kg rests on a rough horizontal floor. A horizontal force of 30 N is applied but the box does not move.

(a) Find the normal reaction. [1]

(b) Given the box is on the point of slipping, find the coefficient of friction. [3]

3.3 A particle of weight W hangs in equilibrium from two light strings making angles 30° and 60° with the horizontal. The tensions are T_1 and T_2 . Given $W = 20$ N, find T_1 and T_2 . [5]

3.4 A particle of mass 6 kg rests on a rough plane inclined at 25° to the horizontal. The coefficient of friction between the particle and the plane is 0.35. A force P acts on the particle **up the line of greatest slope**. Take $g = 9.81 \text{ m s}^{-2}$.

- (a) Find the normal contact force. [2]

- (b) Find the value of P for which the particle is on the point of sliding **down** the plane. [4]

- (c) Find the value of P for which the particle is on the point of sliding **up** the plane. [3]

- (d) Hence state the range of values of P for which the particle remains in equilibrium. [1]

- (e) Explain why the friction force changes direction between parts (b) and (c). [2]