

3.7 Vectors

Name: _____ Class: _____ Date: _____ Total: 16 marks

KEY WORDS vectors 向量 • position vector 位置向量 • scalar product 数量积 • scalar 标量
• magnitude 模长

Objective

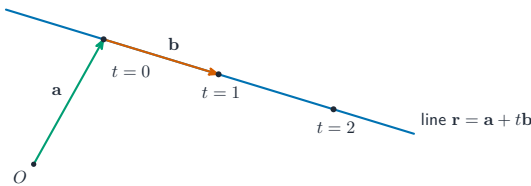
Build the skills to answer exam questions on **vectors** 向量 — magnitude and unit vectors, the **vector equation of a line** 直线向量方程, the **scalar product** 数量积, and the angle between lines.

You must be able to:

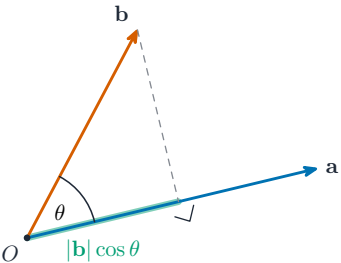
- find magnitude, a unit vector and a displacement $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$
- write and use $\mathbf{r} = \mathbf{a} + t\mathbf{b}$; test for intersection
- use $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$ to find angles / perpendicularity

1 Worked examples

■ Line and scalar product



$\mathbf{r} = \mathbf{a} + t\mathbf{b}$: start at $\mathbf{a}$, add t copies of the direction $\mathbf{b}$



$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$; it is 0 when the vectors are perpendicular

Angle between $\mathbf{a} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$, $\mathbf{b} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$: $\mathbf{a} \cdot \mathbf{b} = 8$, $|\mathbf{a}| = |\mathbf{b}| = 3$, so $\cos\theta = \frac{8}{9}$, $\theta = 27.3^\circ$.

2 Practice

2.1 Find $|\mathbf{v}|$ for $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j} + 12\mathbf{k}$. [2]

2.2 Find $\mathbf{a} \cdot \mathbf{b}$ for $\mathbf{a} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$, $\mathbf{b} = \mathbf{i} + 3\mathbf{j} + \mathbf{k}$. [2]

3 Exam-style questions

3.1 Two vectors are perpendicular when their scalar product is: [1]

- | | |
|---|---|
| A | B |
| C | D |
- A 1
B 0
C the product of their magnitudes
D undefined

3.2 The line l has equation $\mathbf{r} = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} + t \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$.

(a) Find the position vector of the point on l where $t = -3$. [2]

(b) Find the acute angle between l and the direction $\mathbf{d} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$. [4]

3.3 Points $A(1, 2, 1)$, $B(3, 3, 4)$ and $C(2, 0, 3)$ are given.

(a) Find $\overrightarrow{AB}$ and $\overrightarrow{AC}$. [2]

(b) Find angle BAC . [3]
