

3.6 Numerical solution of equations

Name: _____ Class: _____ Date: _____

Total: 13 marks

KEY WORDS iteration 迭代 • converges 收敛 • sign change 变号
 • numerical solution of equations 方程的数值解

Objective

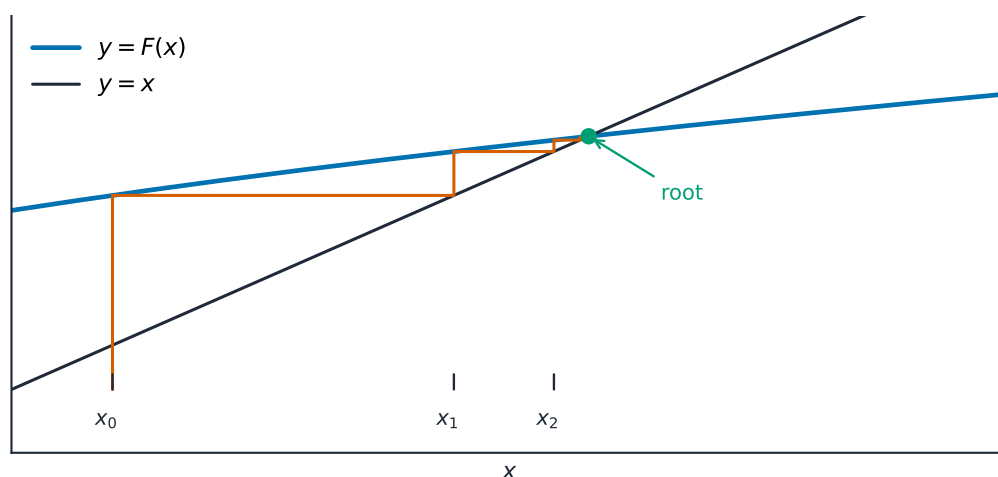
Build the skills to answer exam questions on the **numerical solution of equations** 方程的数值解 in Pure 3 — locating a root by a **sign change** 变号 and finding it by **iteration** 迭代, often after some algebra/calculus.

You must be able to:

- show a root lies in an interval by a sign change
- show an equation rearranges into a given iterative form
- iterate $x_{n+1} = F(x_n)$ to the required accuracy

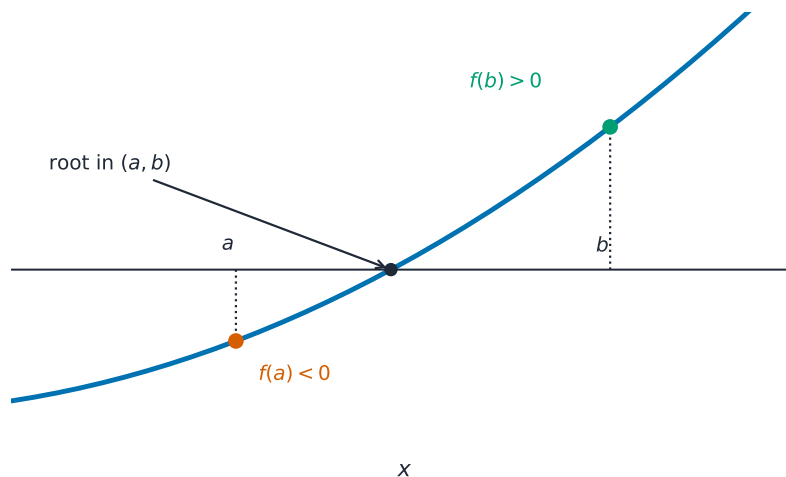
1 Worked examples

■ Iteration to a root



$$x_{n+1} = g(x_n) \quad \cdot \quad \text{converges near root if } |g'(x)| < 1$$

Each step: up to $y = F(x)$, across to $y = x$ — the iteration converges to the root



if f continuous on $[a, b]$ and $f(a)f(b) < 0$, then a root lies in (a, b)

Locate the root first by a sign change

$x_{n+1} = \sqrt[3]{-2x_n - 4.5}$ from $x_0 = -1.2$: $x_1 = -1.281$, $x_2 = -1.247$, ... $\rightarrow$ root ≈ -1.26 (3 s.f.).

2 Practice

2.1 $f(x) = e^x - 3x$. Show that a root lies between $x = 1$ and $x = 2$. [2]

2.2 Rearrange $x^3 + x - 5 = 0$ into the form $x = F(x)$ suitable for iteration. [1]

3 Exam-style questions

3.1 An iteration $x_{n+1} = F(x_n)$ from a suitable start **converges** to a root if the values:[1]

A	B
C	D

A grow without limit

B settle towards a fixed number

C alternate between two values forever

D stay equal to x_0

3.2 The equation $x^3 = 4x + 1$ has a root α near $x = 2$.

(a) Verify there is a root between $x = 2$ and $x = 2.2$. [2]

(b) Show the equation can be written $x = \sqrt[3]{4x + 1}$ and use $x_{n+1} = \sqrt[3]{4x_n + 1}$, $x_0 = 2.1$, to find α to 2 d.p. [4]

3.3 By sketching $y = e^{-x}$ and $y = x$ on the same axes, state the number of roots of $e^{-x} = x$, and give an interval of width 0.5 containing the root. [3]
