

Solutions

Each answer shows the steps, then the **result**.

2.1

- $f(1) = e - 3 = -0.282$, $f(2) = e^2 - 6 = 1.389$; opposite signs
- a root lies between 1 and 2

2.2

- e.g. $x = \sqrt[3]{5 - x}$

3.1 B

- Convergence means the iterates settle towards a fixed value

3.2

(a)

- $f(x) = x^3 - 4x - 1$: $f(2) = -1 < 0$, $f(2.2) = 10.648 - 8.8 - 1 = 0.848 > 0$; sign change
- root between

(b)

- $x^3 = 4x + 1 \Rightarrow x = \sqrt[3]{4x + 1}$; $x_1 = \sqrt[3]{9.4} = 2.110$, $x_2 = 2.113$, $x_3 = 2.114$; $\alpha = 2.11$

3.3

- the decreasing curve $y = e^{-x}$ meets the line $y = x$ exactly **once**, so there is 1 root; $g(x) = e^{-x} - x$ has $g(0.5) = 0.607 - 0.5 > 0$ and $g(1) = 0.368 - 1 < 0$, so the root lies between 0.5 and 1