

3.5 Integration (by parts, substitution, partial fractions)

Name: _____ Class: _____ Date: _____

Total: 16 marks

KEY WORDS substitution 换元积分 • integration 积分 • integration by parts 分部积分
• partial fractions 部分分式 • rational function 有理函数

Objective

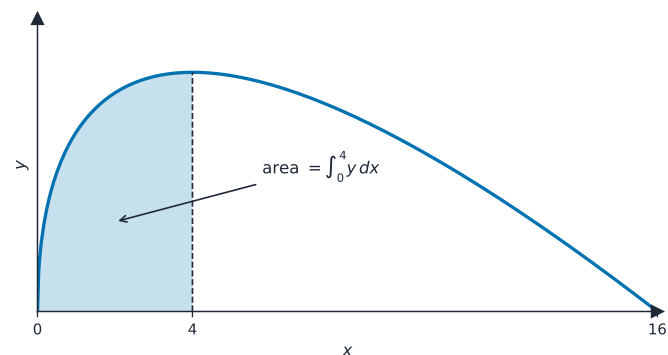
Build the skills to answer exam questions on **integration** 积分 in Pure 3 — the standard integral $\frac{1}{x^2 + a^2}$, **integration by parts** 分部积分, **substitution** 换元积分, partial fractions, and the $\frac{kf'(x)}{f(x)}$ pattern.

You must be able to:

- recognise $\int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$ and $\int \frac{kf'(x)}{f(x)} dx = k \ln |f(x)| + C$
- integrate by parts and by a given substitution
- integrate a rational function via partial fractions

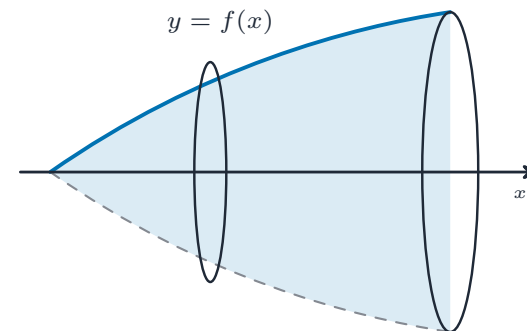
1 Worked examples

- By parts & the log pattern



$$\int_a^b f(x) dx = \text{net signed area between } y = f(x) \text{ and the } x\text{-axis}$$

A definite integral is still an area; Pure 3 adds the methods to evaluate harder ones



rotate the region around the x -axis to sweep a solid

An application of integration: a volume of revolution

$$\int x \cos x dx \text{ — by parts } (u = x, dv = \cos x dx): = x \sin x - \int \sin x dx = x \sin x + \cos x + C.$$

$$\int \frac{2x}{x^2 + 1} dx = \ln(x^2 + 1) + C \text{ (top is the derivative of the bottom).}$$

2 Practice

2.1 Find $\int \frac{1}{x^2 + 9} dx$. [2]

2.2 Find $\int \frac{3x^2}{x^3 + 1} dx$. [2]

3 Exam-style questions

3.1 $\int xe^x dx$ equals: [1]

- A

B
- C

D
- A $xe^x - e^x + C$
- B $xe^x + e^x + C$
- C $\frac{1}{2}x^2e^x + C$
- D $e^x + C$

3.2 Use the substitution $u = 1 + x^2$ to find $\int_0^2 x\sqrt{1 + x^2} dx$. [5]

3.3 (a) Express $\frac{4}{(x)(x + 2)}$ in partial fractions. [3]

(b) Hence find $\int_1^3 \frac{4}{x(x + 2)} dx$, giving an exact answer. [3]
