

3.4 Differentiation (inverse tangent, implicit, parametric)

Name: _____ Class: _____ Date: _____

Total: 14 marks

KEY WORDS quotient 商 • inverse tangent 反正切 • differentiation 微分

Objective

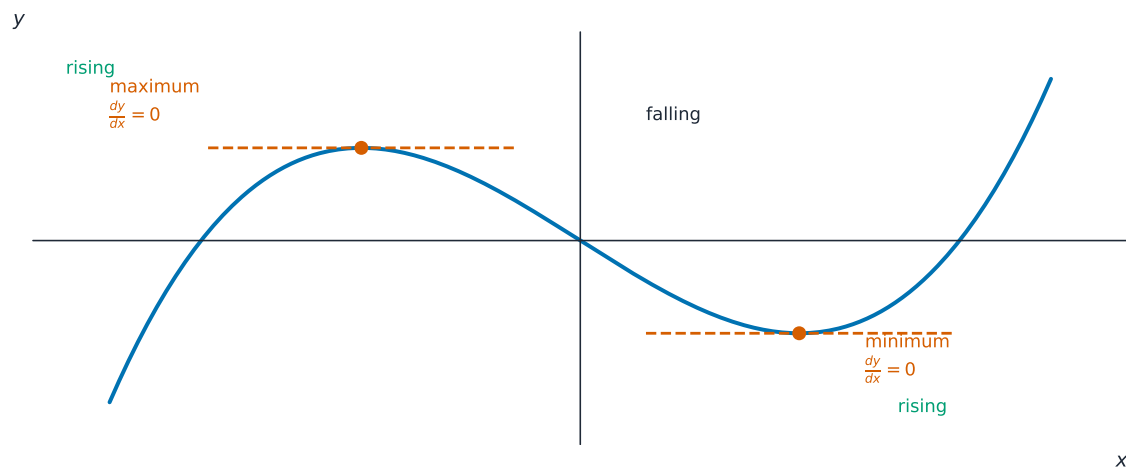
Build the skills to answer exam questions on **differentiation** 微分 in Pure 3 — the Pure 2 rules plus the **inverse tangent** 反正切 derivative, applied to parametric and implicit curves.

You must be able to:

- use $\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$
- apply the product, quotient and chain rules
- differentiate parametric and implicit relations

1 Worked examples

- Mixed rules



stationary points: $\frac{dy}{dx} = 0$ · use $\frac{d^2y}{dx^2}$ or first-derivative sign to classify

Stationary points and gradients still come from differentiating — now with $\tan^{-1}$ and implicit methods too

$$y = \tan^{-1}(2x): \text{chain rule} \rightarrow \frac{dy}{dx} = \frac{1}{1 + (2x)^2} \cdot 2 = \frac{2}{1 + 4x^2}.$$

$$\text{Implicit } e^{2y} = x + y: 2e^{2y} \frac{dy}{dx} = 1 + \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{2e^{2y} - 1}.$$

2 Practice

2.1 Differentiate $y = \tan^{-1}(x^2)$. [2]

2.2 Differentiate $y = x \ln x$. [2]

3 Exam-style questions

3.1 $\frac{d}{dx} \tan^{-1}(3x)$ equals: [1]

- | | |
|---|---|
| A | B |
| C | D |
- A** $\frac{3}{1+9x^2}$
B $\frac{1}{1+3x^2}$
C $\frac{3}{1+3x^2}$
D $\frac{1}{1+9x^2}$

3.2 A curve has parametric equations $x = 1 + 2 \cos t$, $y = 3 \sin t$ for $0 \leq t \leq \pi$. Find $\frac{dy}{dx}$ in terms of t and the gradient at $t = \frac{\pi}{6}$. [4]

3.3 A curve is given implicitly by $x^2y + y^3 = 10$.

(a) Find $\frac{dy}{dx}$ in terms of x and y . [3]

(b) Find the gradient at the point $(1, 2)$. [2]
