

3.1 Algebra (partial fractions, rational-power binomial)

Name: _____ Class: _____ Date: _____ Total: 17 marks

KEY WORDS partial fractions 部分分式 • binomial expansion 二项展开式 • further algebra 代数
• rational function 有理函数 • converges 收敛

Objective

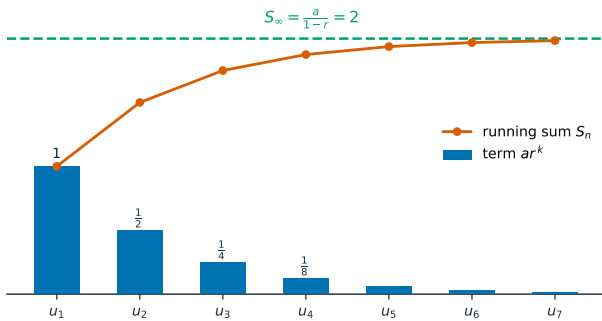
Build the skills to answer exam questions on **further algebra** 代数 — **partial fractions** 部分分式 and the **binomial expansion for a rational power** 分数次幂二项展开式.

You must be able to:

- split a rational function into partial fractions (including a repeated factor)
- expand $(1+x)^n$ for fractional/negative n (for $|x| < 1$)
- combine the two (expand after splitting)

1 Worked examples

■ Partial fractions & rational-power binomial



$$S_n = \frac{a(1-r^{n+1})}{1-r} \cdot S_{\infty} = \frac{a}{1-r} \text{ only if } |r| < 1$$

The rational-power binomial series converges only for $|x| < 1$ — the terms must shrink

$$\frac{x+4}{(x+1)(x-2)} = \frac{A}{x+1} + \frac{B}{x-2}; \quad x+4 = A(x-2) + B(x+1); \quad x=2 \Rightarrow B=2;$$
$$x=-1 \Rightarrow A=-1. \text{ So } \frac{2}{x-2} - \frac{1}{x+1}.$$
$$(1+x)^{1/2} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \dots \text{ for } |x| < 1.$$

2 Practice

2.1 Find the first three terms of $(1+x)^{-1}$. [2]

2.2 Write $\frac{5}{(x-1)(x+4)}$ in the form $\frac{A}{x-1} + \frac{B}{x+4}$. [3]

3 Exam-style questions

3.1 The expansion of $(1 + 2x)^{1/2}$ in ascending powers of x begins: [1]

- | | |
|---|---|
| A | B |
| C | D |
- A** $1 + x - \frac{1}{2}x^2 + \dots$
B $1 + x + \frac{1}{2}x^2 + \dots$
C $1 + 2x - 2x^2 + \dots$
D $1 - x + x^2 + \dots$

3.2 Express $\frac{9x + 5}{(x + 1)(2x - 1)}$ in partial fractions. [4]

3.3 (a) Express $\frac{2}{(1 - x)(1 + 2x)}$ in partial fractions. [3]

(b) Hence find the expansion of $\frac{2}{(1 - x)(1 + 2x)}$ up to the term in x^2 . [4]
